Robert Dalton Smart

NUMERICAL REFRACTION AND DIFFRACTION
FOR COASTAL AREAS.

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by

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SB, Massachusetts Institute of Technology
(1957)

Submitted in partial fulfillment
of the requirements for the degrees of
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ABSTRACT

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Submitted to the Department of Civil Engineering on May 19, 1967, in partial fulfillment of the requirements for the degrees of Master of Science in Civil Engineering and Civil Engineer.

The purpose of this study was to develop a computer system which could be used by engineers in analysis of water wave behavior in coastal areas.

A wave ray tracing program was developed which uses three-dimensional continuous parabolic interpolation for determination of bottom depths and derivatives. Tested on analytical plane beaches this method gave results which agreed within 1 per cent of theoretical results. Tested on a natural beach, the program agreed favorably with graphical methods and numerical methods developed by others.

For study of diffraction patterns both at ends of semi-infinite breakwaters and at breakwater gaps, a program was developed, based on Penney-Price methods, which calculates the coefficient of diffraction at any point in a diffraction zone. Using formulae derived from the Penney-Price methods, a program was developed which calculates directly the direction of travel of a wave at any point in the diffraction zone. The results of this program compare closely with graphical plots of diffraction zones which were completed by manual methods.

These major programs, and a smaller program which locates and traces shorelines across an array of water depths, were joined into one system through a Problem-Oriented Language. This system reads and works with the same raw data available to an engineer in the field performing analysis. It will perform tasks and make calculations as directed by the operator, then output all information needed by the engineer in his analysis.

Either analytical or natural beaches, in feet or fathoms, or combinations thereof may be input to the program. To simulate tidal fluctuations, water level changes may be made at any time during analysis. Breakwaters may be superimposed on the array of bottom depths; and, if the breakwater or its diffraction zones are intersection by a wave during refraction analysis, all ray values at the point of intersection are calculated.

Program output includes, shoreline traces, diffraction coefficients with direction of travel of diffracted waves, and wave ray traces. Values are calculated at given points along the ray including, among other data, direction of wave travel, curvature, coefficient of refraction, and the shoaling factor.

Thesis Supervisor: Frank E. Perkins

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Professor Frank E. Perkins provided many suggestions on how this problem could be solved and offered the author considerable assistance whenever obstacles were encountered. Without his advice this study would not have been as successful as it was. The author wishes to express his sincere gratitude and thanks.

Most importantly, I wish to acknowledge the unfailing moral support and encouragement given by my wife, Betty.

TABLE OF CONTENTS

Page

TERMS AND SYMBOLS

Wherever possible, terms and symbols used in derivatives and text are in accordance with the standards of the Council on Wave Research as presented by Wiegel [1][1]. Hardware and program language requirements prevented use of all symbols in actual programs however and the need for additional variables may have resulted in notation which is in conflict with the standards. Comments within program summaries explain correlation of program to text notation. Some symbols have different meaning in different portions of the text; however, their use in any particular instance should be clear from the text.

Symbols used in text:

b	Length of wave crest between orthogonals	
C	1)	Wave velocity (Phase velocity)
	2)	In a Fresnel Integral, the real portion
C_g	Wave group velocity	
D	Shoaling Coefficient	
d	Depth of water - still level to bottom	
f	Function of one or more variables, e.g. $f(X,Y)$	
g	1)	Acceleration of gravity (32.2 ft./sec.^{2})
	2)	Function of one or more variables, e.g. $g(X,Y)$
H	Wave height	
K_d	Refraction coefficient	

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[1] Numbers in brackets refer to References listed in Section VIII.

K'	Diffraction Coefficient
K	$2\pi/L$
L	Wave length
r	Distance from end of breakwater to point (X, Y)
S	1) Increment distance used when tracing a wave ray
	2) Imaginary portion of a Fresnel integral
T	Wave period
t	Time
X	Horizontal coordinate
Y	Vertical coordinate
Δ	(Delta) change
σ	(Delta) change
η	(eta) surface elevation
π	pi = 3.14159.......
θ	Angular displacement
$-_0$	Subscript "$_0$" refers to deep water value of subscripted property

I. INTRODUCTION

Waves and their effects constitute a major part of civil engineering. Countless books and papers have been written on wave generation, wave theory, wave propagation, and wave effects on coastal areas and coastal facilities. Waves, and especially ocean waves, are already complex in a deep water environment; once the waves enter coastal areas where interaction takes place with a bottom surface and with coastal facilities such as breakwaters, the problems become even more complex.

In shallow water, wave speed is a function of water depth. As waves approach a shoreline, wave crests are bent through the process of refraction to conform to bottom contours. This refraction process causes changes in both wave height and direction of travel, both of which are essential factors in any coastal engineering problem.

Through the process of diffraction, waves and energy are propagated into geometric shadow areas behind breakwaters while reflected waves and energy move into regions beyond the ends of breakwaters. Wave patterns in diffraction zones are a complex of exponentially decreasing amplitude waves spread in an arc in the lee of the breakwater, while sets of waves alternately reinforce and cancel each other in an irregular pattern beyond the breakwater tip.

From the second half of the last century through World War II, these problems were solved by empirical methods and through model studies. However, due to military necessity for the conduct of operations across beaches, extensive research and analysis was conducted during the War, and resulted in major advances which formed the basis of the science as it now stands today.

At present, construction of wave refraction diagrams for use in engineering analysis requires manual drafting techniques such as those presented by Johnson, O'Brien, and Isaacs [2]. However, these methods are slow, tedious, and in many ways subjective. Although there have been various mathematical solutions for special cases of sloping, circular, shelf, and other analytical beaches, these solutions are of limited use to engineers in solving problems of natural beaches.

Penney and Price [3] showed that the diffraction of light is also a solution to the water wave diffraction problem. Using Penney and Price's theory as a basis, various diffraction diagrams have been published showing wave fronts and coefficients of diffraction for standard breakwater problems. Due to the extreme complexity of the mathematics of the problem, various approximations are usually made to simplify the solutions. Results sometimes reflect these simplifications. For engineering studies, diffraction analysis is performed through the use of overlays and tracings based on dimensionless plots which have been published. These plots are available for standard diffraction problems at semi-infinite breakwaters and at breakwater gaps of various widths.

In the interest of good engineering analysis of problems, and in view of the capabilities which are now available to the engineer through the use of electronic computers, a numerical analysis of these problems which is only limited by the basic theory is possible. The basic intent of this study was to develop a program system which would accept as input the same raw data which is available to an engineer; recognize any limitations which the engineer wished to impose on the problem; then carry out whatever sequence of commands the engineer wishes to use in the thorough analysis of any coastal area.

The problem of adapting the refraction process to a computer was first approached by Lt. G. M. Griswold of the U. S. Navy Weather Research Facility [4] in 1963. His efforts and later attempts by others have had varying degrees of success. Early programs often gave erroneous results; later programs worked better but did not calculate all the information needed by engineers; none of the programs were readily suited for use by practicing engineers. Analysis of the refraction problem therefore meant picking up where prior studies had left off.

The major problem involved in refraction analysis is that of simulating bottom topography in a computer. This was the principle difficulty experienced with programs which were developed and tested by Harrison [5].

In view of the success which had been achieved by Snyder [6] at approximating planar curves through a method of continuous parabolic interpolation, it was felt that the same principle, adapted to three-dimensional surfaces, offered an excellent yet simple means of surface fitting for refraction studies. The method would assure continuous quadratic surfaces at any point within the data structure. A method was therefore developed for use of three-dimensional continuous parabolic interpolation in tracing wave rays over a field of varying depths. The method was tested on sloping planar analytical beaches and gave results which agreed within 1% of the actual theoretical values. When tested on a natural beach, it gave results which very closely approximate the graphical procedures now being used. Although more extensive testing is desired, the system developed appears to offer a means of tracing wave rays which is readily suited to use in engineering analysis. The system is of as good or better accuracy than present methods, and yet can offer electronic computer capability for rapid economical solutions.

As best as could be determined, the problem of adapting the diffraction process to a computer has not been approached. Various methods have been developed for determining coefficients of diffraction using the original Penney and Price theories. Without too much difficulty, these methods are adaptable to numerical analysis. For accurate engineering analysis however, it is also necessary to know the direction of wave travel at any given point in a diffraction zone. Although methods are available for calculating wave phases at any point in a diffraction zone, the only method at present to calculate wave direction of travel is through a graphical analysis of many points in an area surrounding the point in question. No means of directly calculating the direction of wave travel at a point in a diffraction zone has been published.

Solutions of the diffraction problem therefore involved adapting existing diffraction methods to a numerical procedure for the computation of diffraction coefficients; and the algebraic analysis of a diffracted wave front to determine its direction of travel then adapting this analysis to a numerical procedure. Results obtained from the analysis and programs are comparable to those achieved through manual and graphical techniques.

The refraction and diffraction programs which were developed were then incorporated along with an algebraic program to locate shorelines into a system which could readily be used by a practicing engineer. A problem-oriented language was used for operation command definition; input is the same basic data available to the engineer in the field; output is the same product information desired by an engineer in any analysis.

Use of this type of program could mean that, given necessary input data, engineering analysis of an area could be completed by one engineer within a matter of minutes rather than in a matter of months as is the case using older methods.

II. REFRACTION ANALYSIS

A. Refraction Principles

The velocity of a wave, C, varies with water depth in accordance with the basic relationship presented by Eagleson and Dean [7] and others

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi d}{L} \qquad (1)$$

where g is the acceleration of gravity, L is the wave length and d the water depth. With a constant period, velocity and wave length decrease as depth decreases. In deep water,

$$\tanh \frac{2\pi d}{L} = 1.00$$

and

$$C_0 = \frac{gT}{2\pi}$$

Looking at the crest of a long wave moving at an angle to the shore, it can be seen that in accordance with equation (1) the deep water portions of the crest move faster than the shallow water portions. This velocity variation causes the wave length to change and the wave to bend toward alignment with the contours. Refraction is the process whereby the direction of motion changes due to a change in wave velocity.

Refraction results in changes in wave height, direction of travel, and change all characteristics of the wave except for its period which still remains constant. The extent of these changes depends on the bottom topography, the wave period, and original deep water direction of

travel. In performing refraction analysis, it is assumed that as a wave travels shoreward, no energy flows laterally along a crest; the energy transmitted between orthogonals to the wave crests will remain constant. The validity of this assumption will be discussed later. Using energy considerations it can readily be shown, Wiegel [8] (pp. 156), Beach Erosion Board [9], and Johnson [10], that

$$H = H_0 \sqrt{\frac{C_{g0}}{C_g}} \sqrt{\frac{b_0}{b}}$$

where H is the wave height. C_g is the group velocity, and b is the orthogonal spacing. The <u>shoaling coefficient</u>, D, is defined by

$$D = \sqrt{\frac{C_{g_0}}{C_g}}$$

which can be evaluated through the use of:

$$C_g = n\ C$$

$$n = \frac{1}{2}\ [1 + \frac{4\pi d/L}{\sinh\ 4\pi d/L}]$$

$$n = \frac{1}{2} \text{ in deep water}$$

This shoaling coefficient gives the ratio of the wave height in water at some shallower depth to the height of the wave in deep water with no refraction effects.

This study however will be more concerned with the derivation of the coefficient of refraction, K_d, defined by:

$$K_d = \sqrt{\frac{b_0}{b}} \tag{2}$$

The coefficient of refraction gives the effect of wave crest bending, or changes in spacing of adjacent orthogonals, on the ratio of local wave height to the deep water wave height.

The basic principle behind all refraction diagrams is Snell's Law:

$$\frac{\sin A1}{\sin A2} = \frac{C1}{C2}$$

where A is the angle between a wave front and its respective bottom contour and C is as given by (1). This law states that, where bottom contours are parallel, the sine of the angle between the wave crest and the bottom contour is proportional to the velocity of the wave. When all contours are straight and parallel it makes no difference whether the change of depth is a continuous slope or a step, the change in wave length is only controlled by the deep water and shallow water wave lengths.

Although equations have been derived for the solution of circular and parabolic beaches by Wiegel [8] (p. 160) and by Pocinki [11], for natural beaches, final wave direction is very dependent on intermediate contours and incremental procedures must be used for solutions.

B. Present Methods of Refraction Analysis

There are two methods in common use today; both are graphical.

The Wave Crest Method uses a graduated scale to plot wave advance from point to point along a crest. Starting in deep water, successive wave crests are plotted until the beach or end of the study area is reached.

The Crestless Method plots wave orthogonals directly by determining shoreward deflections as orthogonals cross successive bottom contours. The amount of deflection is obtained from formulae derived from Snell's Law.

Both of these methods are described in considerable detail in various publications including Johnson, et al [2], Bretschneider [12], Dunham [13], and Beach Erosion Board [9]. In addition, an apparent improvement to the crestless method was developed by Arthur, Munk, and Isaacs [14] which gave closer to theoretical results when tested on concentric circular contours. Each method has its advantages and disadvantages and the method used in any particular situation depends on wave and topographic characteristics, and also the engineer/draftsman's capabilities. Both methods, if applied and used properly, give results which are in reasonable agreement with aerial photographs of study areas as shown by Dunham [13].

However, there are two very basic problems with both systems. First of all, they depend very much on the operator's skill, ability, and judgment. According to Johnson, et al [2], "Since bottom features which are comparatively small in respect to the wave length do not affect the wave appreciably" standard operating procedure is to "use judgement" and "smooth out" irregularities. It is left to the engineer to define "comparatively small" and "affect...appreciably".

As shown by Dunham [13] and Pierson [15], different operators can get different results using the same system, the same operator can get markedly differing results using different systems. In particular, the wave

crest method may smooth out an area of orthogonal convergence and hide potentially dangerous situations, (Pierson [15]).

A second basic problem is that a thorough study of a coastal area is extremely slow, time consuming, and thereby inherently expensive. Given that very small changes in direction or of period can cause extreme changes at a shoreline, it may be necessary to study small increments of wave periods for from 2 to 20 or more seconds over approximately 180 degree arc ranges, for many areas, at various tide stages. To do a reasonably complete job of studying an area can easily require 2 to 3 man months of effort; (Pierson, et al [16]). For complex study areas this time can increase considerably. For smaller low cost facilities or for those in which time is of critical importance, thorough engineering analysis quickly becomes impossible.

C. Previous Numerical Methods Used in Refraction Analysis

In an effort to overcome the limitations and problems of graphical wave-ray diagrams, Lieutenant Gale M. Griswald [4], Then of the U. S. Navy Weather Research Facility, attempted in 1963 to develop a wave-ray tracing program based on the numerical methods originally proposed by Munk and Arthur [17]. His work, together with that of his associates, resulted in programs being written for the CDC 1604, the IBM 7090, and the Bendix G-15D computers, (Griswald [4] and [19]). However, these programs gave erroneous results in some circumstances apparently because of the bottom curve fitting procedures which were used. Results of these programs are shown by Harrison [5].

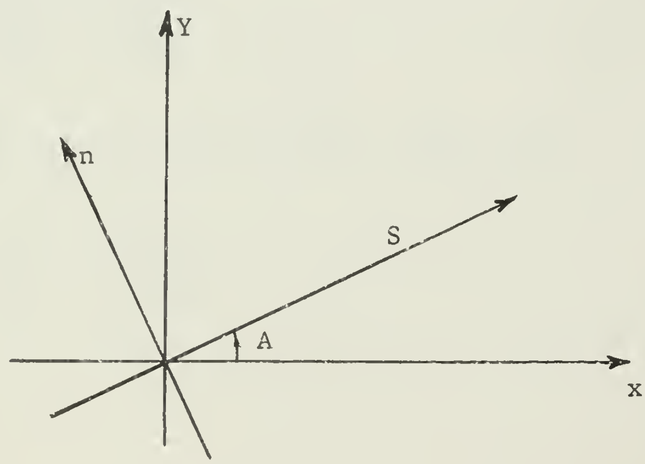
This work was then continued by the Army Coastal Engineering Research Center. Under contract to CERC, Dr. Wyman Harrison, of the

Virginia Institute of Marine Science, developed a method of constructing wave rays which appears operational (Harrison [5]). However, for engineering use this program has many limitations. Because of the linear bottom curve fitting techniques used, it is not capable of calculating a coefficient of refraction as a ray moves shoreward. Independent passes of various programs are required to create input needed by succeeding programs, thereby resulting in rather complex operational procedures. These past numerical analyses and attempts to perform refraction analysis on computers were milestones. However, they did not go far enough towards giving the engineer all the information he needed in a simple efficient manner which he would be willing to use in practice.

The basic approaches and theories behind these programs were sound however and optimum use of their methods and experience formed the basis for this study.

D. Theoretical Considerations for Numerical Refraction

a) Ray Curvature Equations



Ray Notation

Figure 1

As shown in Figure 1, the equations for a ray, S, making an angle, A, with the X axis are

$$\frac{dX}{dt} = \frac{dS}{dt} \cos (A)$$

$$\frac{dY}{dt} = \frac{dS}{dt} \sin (A)$$

Letting $\frac{dS}{dt} = C$, we have

$$\frac{dX}{dt} = C \cos (A) \tag{3}$$

$$\frac{dY}{dt} = C \sin (A) \tag{4}$$

Munk and Arthur [17] apply Fermat's principle[1] to water waves and show that the curvature of the ray depends on the gradient of the velocity field normal to the ray. Thus

$$\frac{dA}{dt} = - \frac{dC}{dn}$$

Substituting $\frac{dS}{dt} = C$ and using $\frac{dA}{dt} = \frac{dA}{dS} \cdot \frac{dS}{dt}$ we get

$$\frac{dA}{dS} = - \frac{1}{C} \frac{dC}{dn} \tag{5}$$

[1] Fermat's Principle states that light travels between two points along that path for which the travel time is a minimum.

We can now express the gradient $\frac{dC}{dn}$ in a more convenient form by using the chain rule

$$\frac{d}{dn} = \frac{\partial}{\partial x}\frac{\partial x}{\partial n} + \frac{\partial}{\partial y}\;\frac{\partial y}{\partial n}$$

Noting from Figure 1 that we can show

$$\frac{\partial x}{\partial n} = -\sin A$$

$$\frac{\partial y}{\partial n} = \cos A$$

and

$$\frac{\partial}{\partial n} = \frac{\partial}{\partial x}\,(-\sin A) + \frac{\partial}{\partial y}\,(\cos A)$$

Defining a new curvature of the ray, K_s, equal to $\partial A/\partial S$ we have from (5) and (6)

$$K_s = \frac{\partial A}{\partial S} = \frac{1}{C}\,(\sin\frac{\partial C}{\partial x} - \cos A\,\frac{\partial C}{\partial y}) \qquad (7)$$

It is possible to express wave velocity and its derivatives from bottom depth and its derivatives.

$$\frac{\partial C}{\partial x} = \frac{\partial d/\partial x}{\partial d/\partial C} \qquad (8)$$

$$\frac{\partial C}{\partial y} = \frac{\partial d/\partial y}{\partial d/\partial C} \qquad (9)$$

One cal calculate $\partial d/\partial C$ from equation (1) as follows:

$$C = \frac{gT}{2\pi} \quad \tanh \quad (\frac{2\pi d}{CT})$$

Setting

$$K' = T/4\pi$$

$$K'' = 2\pi/gT$$

$$C = \frac{1}{K''} \quad \tanh \quad \frac{d}{2CK'}$$

$$d = 2CK' \quad \tanh^{-1} (CK'')$$

$$= 2CK' * \frac{1}{2} [\ln (1 + CK'') - \ln (1-CK'')]$$

$$\frac{dd}{dC} = K' [\ln (1 + CK'') - \ln (1-CK'')] + CK' [\frac{K''}{1+CK''} - \frac{-K''}{1-CK''}]$$

$$\frac{dd}{dC} = K' [\frac{2CK''}{1-(CK'')^2} + \ln (1+CK'') - \ln (1-CK'')] \qquad (10)$$

Given a means of evaluating bottom surface derivatives, equations (3) through (10) can be evaluated to find the curvature of a wave orthogonal at any point and the problem becomes one of tracing this ray of varying curvature from one point to another through a suitable program.

b) Wave Intensity equations

Defining a wave intensity factor, or more accurately a ray separation factor, β, (Beta) by

$$\beta = b/b_0 \qquad (11)$$

where b is the distance between adjacent rays, Munk and Arthur [19] derive the following equation for wave intensity along a refracted ray.

$$\frac{D^2\beta}{DS^2} + p \frac{D\beta}{DS} + q \beta = 0 \qquad (12)$$

$$p(S) = -\cos A \left[\frac{1}{C}\ \frac{\partial C}{\partial X}\right] - \sin A \left[\frac{1}{C}\ \frac{\partial C}{\partial Y}\right] \qquad (13)$$

$$q(S) = \sin^2 A \left[\frac{1}{C}\ \frac{\partial^2 C}{\partial X^2}\right] - 2 \sin A \cos A \left[\frac{1}{C}\ \frac{\partial^2 C}{\partial X \partial Y}\right]$$

$$+ \cos^2 A \left[\frac{1}{C}\ \frac{\partial^2 C}{\partial Y^2}\right] \qquad (14)$$

Factors dd/dC, $\partial C/\partial X$, and $\partial C/\partial Y$ have previsously been defined by equations (10), (8), and (9). To evaluate $\partial^2 C/\partial X^2$, $\partial^2 C/\partial x \partial y$, and $\partial^2 C/\partial Y^2$ we proceed as follows.

$$\frac{\partial C}{\partial X} = \left(\frac{dC}{dd}\right)\left(\frac{\partial d}{\partial X}\right)$$

$$\frac{\partial^2 C}{\partial X^2} = \left(\frac{d^2 C}{dd^2} \circ \frac{\partial d}{\partial X}\right)\left(\frac{\partial d}{\partial x}\right) + \frac{dc}{dd} \circ \frac{\partial^2 d}{\partial x^2}$$

$$\frac{\partial^2 C}{\partial X^2} = -\frac{d^2 d/dC^2}{(dd/dC)^3}\left(\frac{\partial d}{\partial X}\right)^2 + \frac{\partial^2 d/\partial X^2}{dd/dC}$$

Similarly

$$\frac{\partial^2 C}{\partial Y^2} = -\frac{d^2 d/dC^2}{(dd/dC)^3}\left(\frac{\partial d}{\partial Y}\right)^2 + \frac{\partial^2 d/\partial Y^2}{dd/dC}$$

and

$$\frac{\partial^2 C}{\partial X \partial Y} = -\frac{d^2 d/dC^2}{(dd/dC)^3} \circ \frac{\partial d}{\partial Y}\ \frac{\partial d}{\partial X} + \frac{\partial^2 d/\partial X \partial Y}{\partial d/dC}$$

To evaluate $d^2 d/dC^2$ we continue the differentiation of equation (10).

$$\frac{dd}{dC} = K' \left[\ln(1+CK'') - \ln(1-CK'')\right] + CK' \left[\frac{K''}{1+CK''} - \frac{-K''}{1-CK''}\right]$$

$$\frac{d^2d}{dC^2} = K' \left[\left(\frac{K''}{1+CK''} + \frac{K''}{1-CK''} \right) + \left(\frac{K''}{1+K''C} + \frac{K''}{1-K''C} \right) + C \left(\frac{-K^2}{(1+CK'')^2} + \frac{K''^2}{(1-CK'')^2} \right) \right]$$

$$\frac{d^2d}{dC^2} = K' \left[2 \left(\frac{K''}{1+CK''} + \frac{K''}{1-CK''} + C \left(\frac{K''^2}{(1-CK'')^2} - \frac{K''^2}{(1+CK'')^2} \right) \right] \tag{15}$$

Given a means of computing bottom surface derivatives, at any point, equations (13) through (15) can be evaluated. A means of solving (12) for β is then needed. This is discussed in the succeeding section.

Once β is found, the coefficient of refraction is obtained from equations (2) and (11) to give

$$K_d = \beta^{-\frac{1}{2}} \tag{16}$$

E. Program Considerations

a) Basic Program Approach

Solution of refraction problems involves finding some means of tracing a wave ray of known characteristics from deep through shoaling water. As explained earlier, at any point on a natural beach, the wave characteristics are very dependent on the bottom topography between the point and deep water. Procedures which allow for these intermediate effects, must be used for solutions.

Incremental procedures used and tested by Griswald [4] and [19] and by Harrison [5] were basically sound. Therefore, similar techniques were used in the numerical process developed herein.

Using the known wave ray direction of travel and bottom characteristics at a given point, the ray curvature can be calculated using equation (7). Using the initial direction of travel, and knowing the curvature, an estimated location of the ray some incremental distance ahead can be determined. At this estimated location, it is possible to calculate a new ray curvature which can then be averaged with the curvature at the initial point. Using this average curvature, one can then revise the estimated new location of the ray. At this newly estimated location, the curvature can again be calculated, a new average curvature determined, and the process can continue, until the change in curvature between successive position estimates reduces to within an acceptable limit.

It is possible to calculate all desired wave ray information at this new point including the direction of travel. From this new point, the next increment of advance can be determined using the same iterative procedures. The process can continue until the wave ray runs up onto a beach, strikes a breakwater zone, or goes beyond the limits of the array.

The program developed includes procedures which automatically check for shorelines, array limits, and intersection with a breakwater or a breakwater diffraction zone.

To save unnecessary calculations, it is desirable to have the wave ray advance rapidly, using long intervals in deep water where bottom effects are small, and slowly, using shorter intervals, in shallow water where bottom effects are more important. The ray trace process therefore includes a scheme whereby, if a long interval is specified for deep water calculations, the interval will be reduced in successive stages as the ray curvature increases.

These processes and the manner in which they are used in the program are explained in detail in Section IV and Appendix B.

b) Depth matrix vs. velocity matrix

Prior numerical methods for making wave-ray computations have been based on using an initial computer program to change an original depth array into a wave velocity array; then, as necessary, interpolating and deriving velocity values and derivatives from this new velocity matrix for direct input into the velocity/ordinate equations, such as (7), (13), and (14). Making this change to a velocity matrix decreases the amount of calculation needed during the processes of tracing a ray and calculating refraction coefficients. It does this by eliminating the use of chain-rule

multiplication to convert velocity/depth and depth/ordinate derivatives into the necessary velocity/ordinate derivatives.

However, a depth matrix is used in these programs for the following reasons:

(a) An engineer could use mixes of wave periods during the analysis of an area without going through the lengthy procedure of resubmitting his original depth matrix and converting it to a new velocity matrix which corresponds to the changed wave period.

(b) Water levels can be readily changed to simulate tidal fluctuations.

(c) During possible expansion of the programs for further use in coastal analysis beyond refraction/diffraction studies, a depth matrix should have fewer limitations than a velocity matrix.

c) <u>Interpolation surfaces</u>

Various schemes for representing the bottom topography were tested during past analysis of the refraction problem by Harrison and Wilson [5].

A "forced-cubic" interpolation scheme was originally used by Griswold-Mehr which derived a cubic surface of best fit to the velocity values at 12 grid intersections. This surface was forced to pass through the 4 values closest to the point in question and was the cubic of best fit for the next eight closest eight grid intersections. As a cubic, it permitted taking second derivatives for use in wave intensity calculations. When this scheme was tested by Harrison and Wilson [5] it was found to give completely erroneous results in some cases of a wave ray approaching a beach at a shallow angle.

A "quadratic-interpolation" surface which was also tested by Harrison derived a quadratic surface of best fit to 12 grid intersections. This scheme gave good results except when tested within 2 grid units of the shoreline. Here it was found to give excessive curvatures.

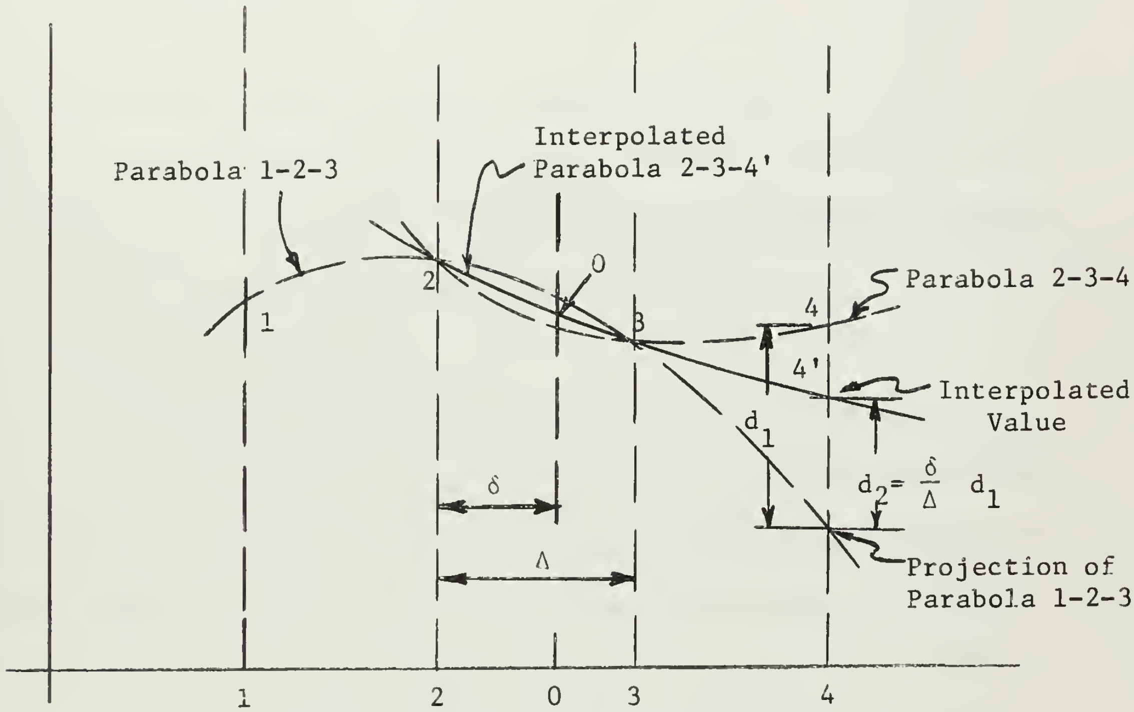
A "linear-interpolation" surface was then developed by Harrison and Wilson, and was based on the assumption that velocity values in a grid cell could be represented by a plane. This scheme gave good results up to within one grid unit of the shoreline but, as a linear surface, did not allow calculation of the derivatives which are needed for wave intensity calculations.

For use by engineers, it was felt that calculation of wave intensity should be a minimum requirement of any program; this necessitates use of at least a quadratic surface. In view of the recent experience of others, a quadratic surface of best fit using least squares and possibly with heavily weighted central grid values appeared to be a logical method. However, this surface would not be continuous as the ray moved from one grid to another. Munk and Arthur [17] point out that for successful solution of equation (5) a continuous surface is necessary.

It was found that excellent results at deriving continuous planar curves using parabolic interpolation had been achieved by Synder [6] in problems not related to refraction.

Based on the geometrical device of linearly transforming the equation of a parabola across an interpolation interval, a smooth curve can be developed which is continuous with no abrupt changes in slope even at data points. The function and its derivatives at the end of one interval are the same as at the start of the next.

The method was developed as a compromise between simple interpolation with a large input requirement and a more complex interpolation scheme with longer computation time. When tested on long segments of a sine wave the method gave results which were almost as good as a cubic equation. Continuous parabolic interpolation appears to offer a curve fitting scheme that is little more difficult than for a simple parabola yet is actually a "third degree" equation. Its principal advantage however is the continuous nature of the function which results.



Continuous Parabolic Interpolation

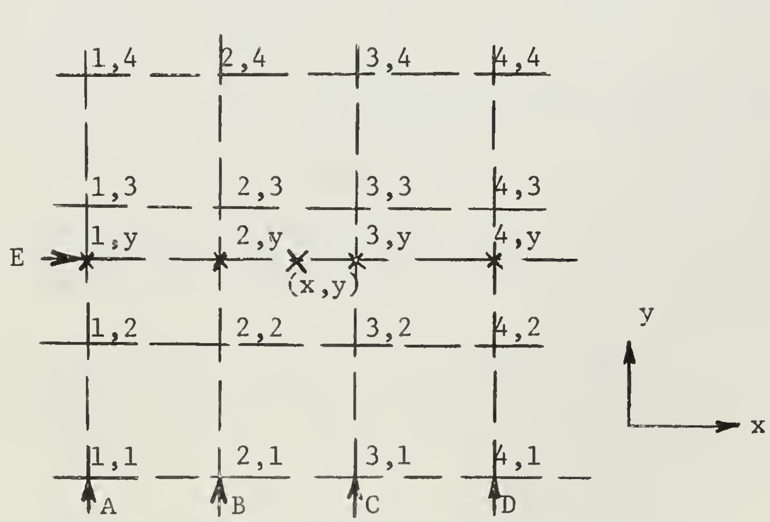
Figure 2

Continuous parabolic interpolation works as shown in Fig. 2. Given 4 data points, 1, 2, 3 and 4, the problem is one of finding the value at abcissa 0 of a smooth continuous curve which passes through all 4 data points. In order to calculate the value and slope at abcissa 0, a parabola of the form $ax^2 + bx + c$ is drawn through points 1, 2, and 3 and its projected value at the abcissa of point 4 is determined. An imaginary

point, 4', is located by linearly interpolating the difference between the projection of parabola 1-2-3 and the actual point 4 value proportional to the advance of point 0 from point 2 to point 3. A parabola through the imaginary point, 4', is used for calculating the curve value and its derivatives at point 0. This interpolated parabola is only valid at point 0. It can readily be seen however that as point 0 moves from point 2 to point 3 a smooth transition is made from parabola 1-2-3 to parabola 2-3-4 and a smooth trace of intermediate data values and derivatives is generated.

Although only used in the past for curve fitting, it was felt that through the use of multiple parallel and orthogonal parabolas the same basic scheme could be adapted to definition of a 3-dimensional surface. Just as the scheme generated smooth curves in one plane it should generate smooth surfaces in 3 dimensions.

The simplicity of the interpolating parabola method, especially when performed on a computer, and the fact that it met the other refraction criteria of providing smooth curves and continuous derivatives at all points made it a best choice for a first try at surface fitting.



Continuous Parabolic Interpolation
for Surface Fitting

Figure 3

For surface fitting, continuous parabolic interpolation was used as shown in Fig. 3. Using the parabolic interpolation techniques just described, 4 independent parabolas were used along vertical planes at A, B, C, and D to calculate d, $\frac{\partial d}{\partial Y}$, and $\frac{\partial^2 d}{\partial Y^2}$ for each of points (1, Y), (2, Y), (3, Y), and (4, Y). A parabola was then interpolated along plane E to determine (X, Y) values. One "E" parabola based on 4 depth values gave d, $\frac{\partial d}{\partial X}$, and $\frac{\partial^2 d}{\partial X^2}$ at (X, Y). Another interpolated parabola based on the $\frac{\partial d}{\partial Y}$ values gave an interplated $\frac{\partial d}{\partial Y}$, and $\frac{\partial^2 d}{\partial X \partial Y}$ for (X, Y) and, similarly, one based on the 4 $\frac{\partial^2 d}{\partial Y^2}$ values gave $\frac{\partial^2 d}{\partial Y^2}$ for (X, Y).

Examination reveals that identical results would be obtained by reversing the X and Y order and first performing 4 interpolations along parallel X planes and the final interpolations along a Y plane. To confirm this fact orthogonal tests were run on real data samples; as expected, both tests yielded identical results for (X, Y) and all its derivatives.

By geometry, it is readily possible to reduce the interpolation procedures described above to simple functions. Using the notation of Fig. 2, but with the 4 equally spaced data values designated A1, A2, A3, and A4; and with D defined as δ/Δ, which is always measured from A2 towards A3; one can solve for the value of A0.

$$A0 = A2 + (D/2)\,(A3 - A1) - (D^2/2)\,(A4 - 4A3 + 5A2 - 2A1) + (D^3/2)\,(A4 - 3A3 + 3A2 - A1)$$

Its first and second derivatives respectively are

$$A0' = (A3 - A1)/2 - (D/2)\,(A4 - 5A3 + 7A2 - 3A1) + (D^2)\,(A4 - 3A3 + 3A2 - A1)$$

$$A0'' = A3 - 2A2 + A1 + (D)\,(A4 - 3A3 + 3A2 - A1)$$

These equations used as functions in a computer program greatly facilitate the solution of continuous parabolic interpolation problems.

d) Solution of the Wave Intensity formula

Various methods for solving the wave intensity formula equation (12)

$$\frac{D^2\beta}{DS^2} + p\,\frac{D\beta}{DS} + q\,\beta = 0$$

are discussed by Munk and Arthur [17]. These methods include:

(a) Using average constant values for p and q within each interval. The equation can be solved and the incremental solutions joined in a continuous curve. This solution appeared complex and not readily suited for incorporation into the other refraction programs.

(b) A "WKB" method involves transformation to forms used in quantum mechanics. This method appeared more complex than (a) preceeding and was ruled out in view of the simpler methods which were available.

(c) Analogue computer methods could be used to solve this problem but were ruled out as beyond the scope of the problem.

(d) Kelvin's method involves approximating the integral curve by fitting together circular arcs. The method is readily adaptable to the associated refraction programs. This method was tested by Griswold [4] to calculate wave intensity for an analytic field of wave velocity values. Wave intensity was then compared with actual theoretical values. The greatest deviation from theoretical was 1.6 per cent.

In addition, Griswold [4] used a simple finite difference method. Using 3 successive values of β, designated BN, B1, and B2, at equal spacing, S, along a curve, one can find by geometry

$$\frac{d^2\beta}{dS^2} \cong (BN - 2B1 + B2) \; / \; (S^2) \qquad (17)$$

$$\frac{d\beta}{dS} \cong (B2 - BN) \; / \; 2S \qquad (18)$$

Substitution of (17) and (18) into (12) and solution of the resulting equation gives

$$B2 = \left(\frac{4 - 2qS^2}{2 + pS}\right) B1 + \left(\frac{pS - 2}{2 + pS}\right) BN \qquad (19)$$

Application of this formula requires that, for three equally spaces points, we know B1, p, and q at the center point, and that we know BN at the initial point. The formula calculates B2 at the third point. This method requires no iterations, and simply involves projecting one Beta value ahead to the next succeeding point, B2. After moving down the ray trace, B2 is then used for the next B1 value and B1 becomes BN. This method was also tested by Griswold [4] against the same analytic field used for the Kelvin method, and was found to give errors of less than 2.2 per cent.

In view of the much greater simplicity of the finite difference method and the fact that it appeared to give results almost as good as the far more complex, Kelvin's method, this method was used in the refraction program.

F. <u>Tests applied to Ray Trace Program</u>

The theory of the Ray Trace program was tested using two methods.

First, it was tested on analytical beaches of uniform slope from deep water up and onto a beach. These tests could be readily verified by means of Snell's Law:

$$\frac{\sin A1}{C1} = \frac{\sin A2}{C2} \tag{20}$$

and a Beta expression derived from Snell's Law which is given by Munk and Arthur [17]

$$\beta = \frac{\sin A2}{\sin A1} \tag{21}$$

One of these tests, which could be considered a typical case is shown in Appendix D. This test shows a wave ray on a plane beach varying in depth from 60 feet to zero. The ray has an initial angle of 45 degrees and a period of 4 seconds. Its error analysis, based on equations (20) and (21) is shown in Table 1. This and similar tests on plane beaches gave results which agree within 1 percent of the theoretical values for both direction of travel and Beta value.

Table 1.

Analysis of Refraction on a Plane Beach

Data Points are from Appendix D

Point	Wave Speed	Test Angle	Theoretical Angle	Angle Error	Test Beta	Theoretical Beta	Beta Error
1	20.49	45.00	45.00	0 %	1.000	1.00	0 %
10	19.97	46.40	46.43	0.06%	1.024	1.024	0.0 %
20	18.41	50.52	50.55	0.06%	1.091	1.092	0.09%
25	17.15	53.68	53.70	0.04%	1.139	1.140	0.09%
30	15.11	58.56	58.57	0.02%	1.206	1.207	0.08%
35	11.51	66.59	66.60	0.02%	1.297	1.298	0.08%
39	5.19	79.71	79.68	0.04%	1.390	1.391	0.07%

Secondly, the method was tested on a section of natural beach at Virginia Beach, Virginia. This test was selected due to the fact that previous refraction methods were tested on this beach by Harrison and Wilson [5], and results achieved could therefore be tested both against graphically constructed wave rays and against other computer programs.

The most accurate method developed by Harrison and Wilson [5] for this beach involved using interpolation with a linear surface of best fit. Therefore, the results of the continuous parabolic interpolation tests are compared to the linear surface method and the more common graphically constructed rays. The comparison is shown on Fig. 4.

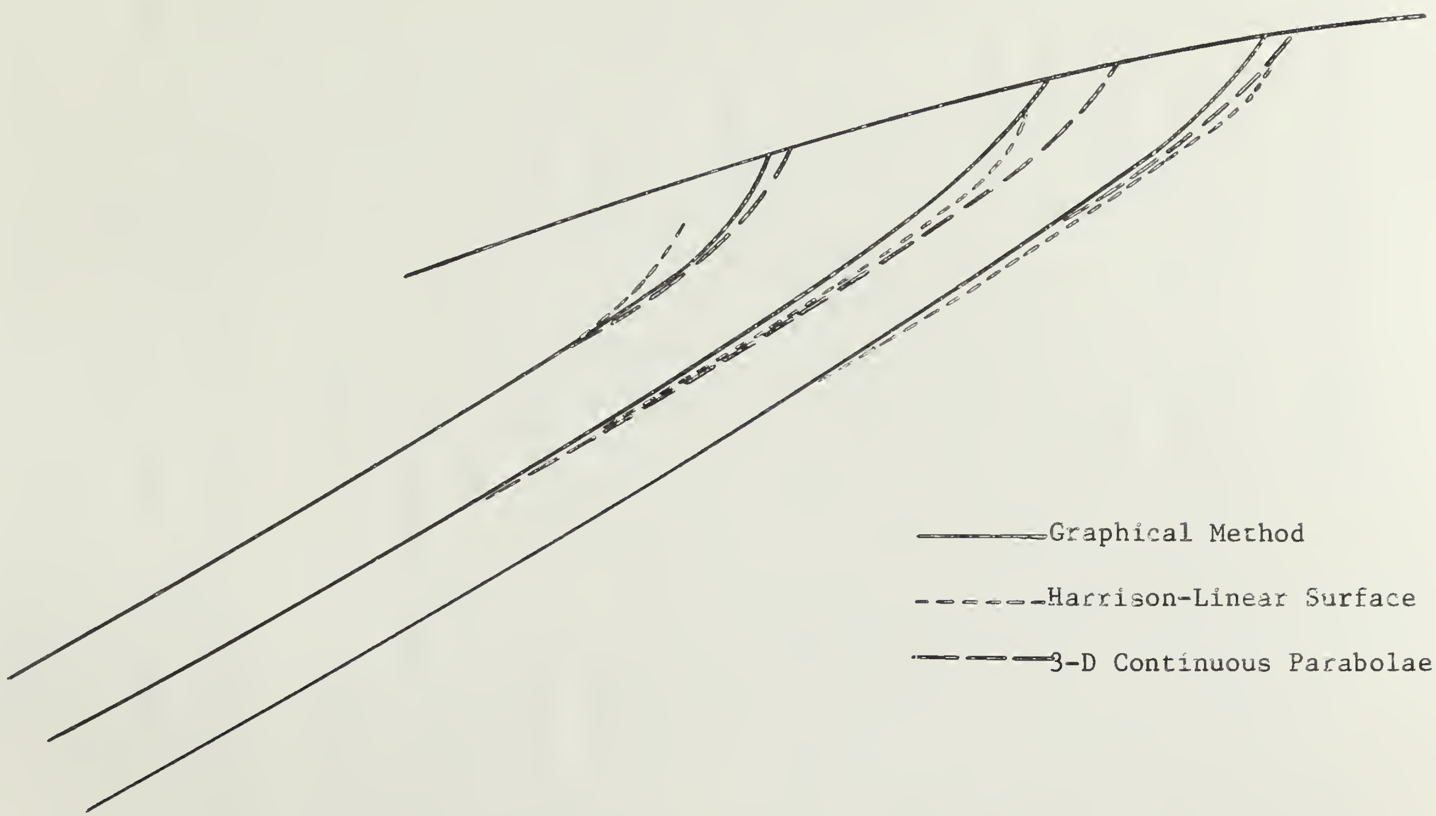
The accuracy of these comparisons was limited by the fact that the input data used by Harrison was not available. It was necessary to estimate input data and his solutions from a 3" by 4" drawing. This could have lead to some differences in results but it was felt that data were suitable for initial testing.

These limited results make it appear that the method developed is comparable to the graphical scheme and linear surface method.

It is also important to remember that none of the methods can be verified under natural conditions for reasons discussed elsewhere in this report. Although graphical methods are most commonly used in practice today, different graphical methods would give different results, different draftsmen using the same method would give different results.

One cannot say with certainty which of the three methods most closely approximates what would actually happen at this beach.

In addition, many specialized minor tests were given the refraction program to assure that special features such as the breakwater intersection check and the gradual reduction of the increment interval do in fact



Refraction Test - Natural Beach

Virginia Beach

Figure 4

work as programmed. These tests gave what amount to Yes/No results. These tests are not enumerated here in view of their lack of significance once they work as desired.

G. Program Limitations

a. Limitations of the Numerical Methods

This program is limited in various ways as a result of the numerical methods used. Wave ray tracing is based on tracing a wave ray using a series of finite intervals. At any point on a natural beach, wave characteristics are dependent on bottom topography between that point and deep water. Any finite interval techniques used may skip over intermittent bottom effects. Extensive testing is needed to determine the effects of this interval technique. When various intervals were tested on the programs developed, the effects were very small; but more testing is needed especially on complex bottoms.

This program is still limited by the ability of parabolic interpolation to accurately represent the bottom surface. In cases of extremely complex bottom topography, it is necessary that grid sizes be small enough that bottom features can be reasonably well represented by a quadratic surface over a span of 2 grid units. This means that over relatively smooth surfaces, large grids are possible; over an irregular bottom, smaller grids should be used.

The solution of equation (12) is obtained by a finite difference scheme. Although this scheme has been tested against analytical beaches by Griswold [4], it should be tested against natural beaches providing some suitable means of verifying results can be obtained.

b. Theoretical Limitations

Most of the same original basic assumptions which were involved in the graphical construction of refraction diagrams are still present in the numerical methods. No new theory has been added; the final results are still limited by the accuracy of this theory.

Refraction Theory is based on the assumption that waves are long crested. Short crested waves, especially if they cross contours at a high angle of incidence, will have refraction coefficients which differ from theoretical, Wiegel (8, p 172).

Although no limits are set, beach slope, roughness, and reflections can effect refraction on natural beaches because of their effect on energy distribution assumptions used in refraction analysis.

The assumption that no energy travels laterally along a crest does not hold when orthogonals bend sharply or form a caustic envelope. A certain amount of energy is transmitted across orthogonals in cases of complex bottoms and where sizeable variations in wave height occur along a crest. Waves do not actually become infinitely high as indicated by a caustic envelope; sometimes they peak and break but usually only a chaotic appearance results according to Pierson, [15]. Similarly, crossed orthogonals may indicate a severence of a wave train and subsequent crossed wave crests, but this is an area which is still full of uncertainties.

Much is still left to the engineer's experience and judgement. There is no precise method for smoothing observed contours to that point at which they will still represent wave refraction but where neglect of minor bottom irregularities is justified. In all cases, accuracy of output is limited by accuracy of the depth data available for input.

III. DIFFRACTION

A. Principles of Diffraction

a) General

Diffraction is the process whereby wave energy is transferred laterally along a wave crest and thereby propagated into areas of geometric shadow behind impervious barriers. It is similar to the diffraction of light that takes place when light is transmitted sideways behind an object so that the area behind the object is slightly lit. Sound and electromagmetic waves experience similar effects.

Diffraction can best be explained by Huygen's Principle which states that each point of a wave front is a source of energy and that it radiates outward equally in all foreward directions (Russell and MacMillan [18]). The wave motion at any point is the sum of the motions induced by all energy sources behind it. In this manner, the "end" of a wave which has been interrupted by a breakwater will act as a source of energy and, in the lee of the breakwater, will spread out in a circular arc of exponentially decreasing amplitude, (Wiegel [8], p. 100). That wave which is reflected off the face of the breakwater also has an "end" which is a source of energy for those regions beyond the breakwater tip. This results in two sets of waves, one advancing and one radiating, which alternately reinforce and cancel each other so as to cause very irregular wave heights in this region beyond the tip.

As a wave approaches a gap, diffraction takes place in both directions as it passes through the gap and two sets of radial waves result from the reflected waves. Addition of the advancing waves and two radial waves in the lee of the breakwater results in an extremely complicated pattern.

Penney and Price [3] show that the distribution, amplitude, and phase of surface waves on a sheet water of any uniform depth can be described by a complex wave function, F(X, Y). Through the use of Sommerfeld's solution for the diffraction of light, polarized in a plane parallel to the edge of a semi-infinite screen, they are able to resolve the water wave diffraction problem.

The complete solution of the water wave diffraction problem has been given in considerable detail by Penney and Price [3], and has since been summarized in various forms by many authors, including Putnam and Arthur [20], Weigel [21] and [8](p 180), and Bretschneider [12]. The reader is referred to these publications for the details and reasoning behind equations which are included in summary form only in this study.

b) Semi-infinite breakwater

Penney and Price [3] show that the equation for the surface elevation, n, in cylindrical coordinates (see Fig. 4) can be stated as

$$n = \frac{AikC}{g} e^{ikCt} \cosh (kd) F(r,\theta)$$

where A is a constant and $k = 2\pi/L$. $F(r,\theta)$ is a function which must satisfy

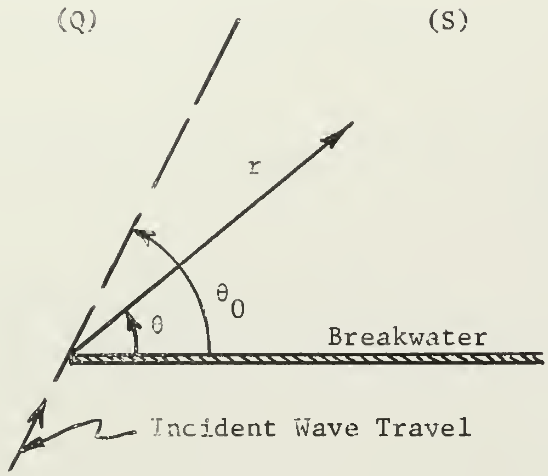
$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2} + k^2 F = 0 \qquad (22)$$

The diffraction coefficient, K', is defined as the ratio of wave height in the area affected by diffraction to the incident wave height and is given by the modulus of $F(r,\theta)$ or

$$K' = |F(r,\theta)|$$

In addition, the wave pattern or phase of the waves can be determined from the argument of $F(r,\theta)$ which is denoted by arg $F(r,\theta)$.

The solution of the diffraction problem is therefore the solution of equation (22), subject to the boundary condition that the normal component of the fluid velocity is zero along the breakwater.



Notations for Semi-infinite Breakwater

Figure 4

Using the notation of Fig. 4, and defining

$$\sigma = 2\sqrt{\frac{kr}{\pi}} \sin\frac{1}{2}(\theta-\theta_0) \qquad (23)$$

$$\sigma' = -2\sqrt{\frac{kr}{\pi}} \sin\frac{1}{2}(\theta+\theta_0) \qquad (24)$$

Penney and Price show that in region "S" (i.e., $0 \leq \theta \leq \theta_0$)

$$F(r,\theta) = f(\sigma)\; e^{-ikr\cos(\theta-\theta_0)} + f(\sigma')\; e^{-ikr\cos(\theta+\theta_0)} \qquad (25)$$

and in region "Q" (i.e., $\theta_0 \leq \theta \leq (\theta_0+\pi)$)

$$F(r,\theta) = e^{-ikr\cos(\theta-\theta_0)} - f(\sigma)\, e^{-ikr\cos(\theta-\theta_0)}$$

$$+ f(\sigma')\, e^{-ikr\cos(\theta+\theta_0)} \qquad (26)$$

where $f(\sigma)$ is defined by

$$f(\sigma) = \frac{1}{2}(1+i)\int_{-\infty}^{\sigma} e^{-\frac{1}{2}\pi i u^2}\, du \qquad (27)$$

Equation (27) can be evaluated through the use of Fresnel Integrals which are defined by

$$C - iS = \int_0^{\sigma} e^{-i\pi u^2/2}\, du$$

or alternately,

$$C(\sigma) = \int_0^{\sigma} \cos\frac{1}{2}\pi u^2\, du \qquad (28)$$

$$S(\sigma) = \int_0^{\sigma} \sin\frac{1}{2}\pi u^2\, du \qquad (29)$$

Substitution into (27) gives

$$f(\sigma) = \frac{1}{2}\left[(1 + C + S) - i\,(S-C)\right]$$

Using the fact that

$$f(-\sigma) = 1 - f(\sigma)$$

we can define

$$f(-\sigma) = U(\sigma) + iW(\sigma) \tag{30}$$

where

$$U = \frac{1}{2}(1 - C - S) \tag{31}$$

$$W = \frac{1}{2}(S - C) \tag{32}$$

Setting

$$f = e^{-ikr\cos(\theta-\theta_0)} f(\sigma) \tag{33}$$

$$g = e^{-ikr\cos(\theta+\theta_0)} f(\sigma') \tag{34}$$

$$f(-\sigma) = U1 + i\,W1 \tag{35}$$

$$f(-\sigma') = U2 + i\,W2 \tag{36}$$

it can be shown algebraically that

$$\begin{aligned} f = {} & U1\cos(kr\cos(\theta-\theta_0)) + U2\cos(kr\cos(\theta+\theta_0)) \\ & + W1\sin(kr\cos(\theta-\theta_0)) + W2\sin(kr\cos(\theta+\theta_0)) \end{aligned} \tag{37}$$

$$\begin{aligned} g = i\,\{ & W1\cos(kr\cos(\theta-\theta_0)) + W2\cos(kr\cos(\theta+\theta_0)) \\ & - U1\sin(kr\cos(\theta-\theta_0)) - U2\sin(kr\cos(\theta+\theta_0))\} \end{aligned} \tag{38}$$

Returning to our original equations it is seen from (25) that in region S

$$F(r,\theta) = f + g \tag{39}$$

while in region Q, (26) gives

$$F(r,\theta) = e^{-ikr\cos(\theta-\theta_0)} - f + g \tag{40}$$

In either case, $F(r,\theta)$ can be expressed as

$$F(r,\theta) = A + i\,B \tag{41}$$

hence

$$K' = |F(r,\theta)| = (A^2+B^2)^{1/2} \tag{42}$$

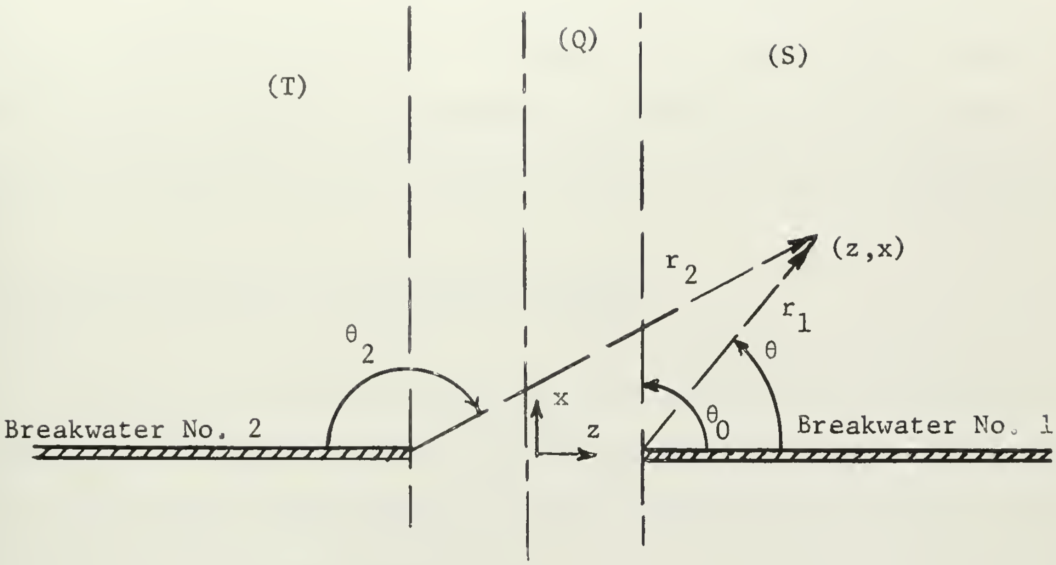
and the phase difference, ϕ, can be expressed as

$$\phi = \tan^{-1}(B/A) \tag{43}$$

Therefore by evaluating σ and σ', solving for C and S in (28) and (29), and making the transformations indicated by equations (31) through (43), we are able to solve for the diffraction coefficient and phase difference at any point in the diffraction zone.

c) Breakwater Gap

Using the following notation



Breakwater Gap Notation

Figure 5

Penney and Price [3] also show that for waves entering a breakwater gap at right angles ($\theta_0 = 90^0$) the diffraction pattern is described by

$$F(r,\theta) = f_1 + g_1 - f_2 + g_2 \qquad \text{for region S} \qquad (44a)$$

$$F(r,\theta) = -f_1 + g_1 + f_2 + g_2 \qquad \text{for region T} \qquad (44b)$$

$$F(r,\theta) = e^{-ikx} - f_1 + g_1 - f_2 + g_2 \qquad \text{for region Q} \qquad (44c)$$

where the subscript indicates the breakwater wing from which the function is measured.

This solution is only valid, however, for waves entering normal to the gap. Upon entering a gap at an oblique angle, a given wave

reaches one tip first and diffracts there before the remainder of the wave reaches the other tip where it will also diffract. Therefore the two wave patterns are out of phase unless the wave crest happens to reach the second tip an exact multiple of wave periods after reaching the first. Equations (44) do not, in general, give valid results with oblique waves. However, an approximate solution for oblique waves is possible through the use of techniques which will be discussed later.

B. Present Methods of Diffraction Analysis

It is apparent that the solution of the diffraction equations is a lengthy procedure when done by manual methods. Fortunately, however, they can be solved independently of wave speed and wave period; and solved in terms of wave length, L.

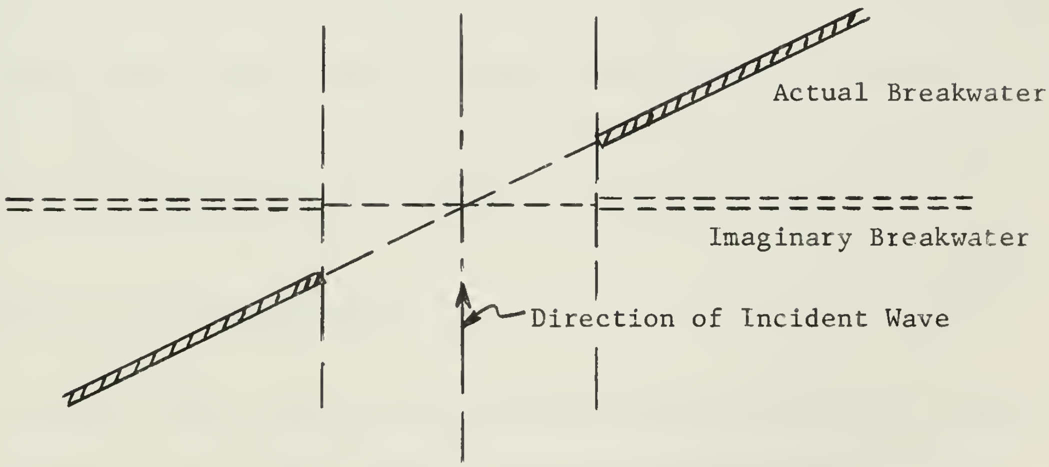
When plotted in terms of X/L and Y/L coordinates, diffraction charts become dimensionless plots of diffraction coefficients which can then be applied to any water wave diffraction problem in water of uniform depth regardless of wave period or wave speed. These solutions have been completed and the results have been included with virtually every diffraction publication, including Wiegel [8](p. 180), Bretschneider [12], and Beach Erosion Board [9]. For the engineer in the field, it then becomes simply a matter of picking one of these dimensionless plots, expanding or reducing the scale so as to fit his particular situation, and with the use of a properly scaled overlay, analyze the particular diffraction situation at hand. Different overlays can be made to represent each wave period to be studied.

The same procedure applies for studying diffraction at breakwater gaps. In these cases, the ratio of gap width, b, to wave length constitutes a second independent parameter; hence, a series of dimensionless

plots are required. These have been published by many authors, including Johnson [24], and Wiegel (3, p 189), for differing b/L ratios from 1 to 5. Beyond 5 wave lengths it is felt that the 2 breakwater tips which form the gap are far enough apart that they can be considered independent for all practical purposes.

Plots of breakwater gaps for waves approaching at an angle are available (Weigel (8, p 193) and Johnson [10]) based on the theories of Morse and Rubenstein, which were developed by Carr and Stelzride [25]. The calculations for these plots are based on series of Mathieu functions which give exact theoretical solutions for incident waves of any angle of approach.

In addition, Johnson [24] showed that a modified gap width as shown in Fig. 6 could be substituted for the actual gap and results achieved which compare very closely to the exact theory of Morse and Rubenstein. This modified gap solution allows the use of Penney and Price equations which were developed earlier.



Modified Breakwater Gap
Wave Approaching at Oblique Angle

Figure 6

In areas sufficiently removed from the breakwater, simplifying approximations can be made for some terms of the various formulae. These approximations have been verified experimentally by Putnam and Arthur [20]. However, in regions close to a breakwater a complete solution is needed.

In addition to Putnam and Arthur [20], diffraction theory has been verified experimentally by Blue and Johnson [26]. Their experiments showed that actual amplitudes agreed very closely with theory in the lee of breakwaters and were somewhat less, but within 10 percent of, theoretical beyond the breakwater tip in the unsheltered zone. This results in generally conservative results, i.e., overdesigned facilities, if the theoretical results are used.

In addition to coefficients of diffraction, it is also necessary to know the direction of wave travel at any given point in a diffraction zone. From equation (43), it is readily possible to calculate the phase difference between the diffracted wave and the wave front of the original incident wave. With this as a basis, it is possible to calculate a number of phase differences within a zone, then by connecting zones of equal phases determine actual wave crest positions and direction of travel. However, no published method has been found which would directly give a direction of wave travel for a given point.

C. Previous Numerical Methods

No publications were found concerning numerical analysis of water wave diffraction problems. However, the present formulation of the diffraction problem is readily susceptible to calculation of coefficients of diffraction in their present form, hence it is possible that

portions of the problem have been done by computer as an aid to graphical analysis but were not considered worthy of publication.

D. Theoretical Considerations for Numerical Diffraction

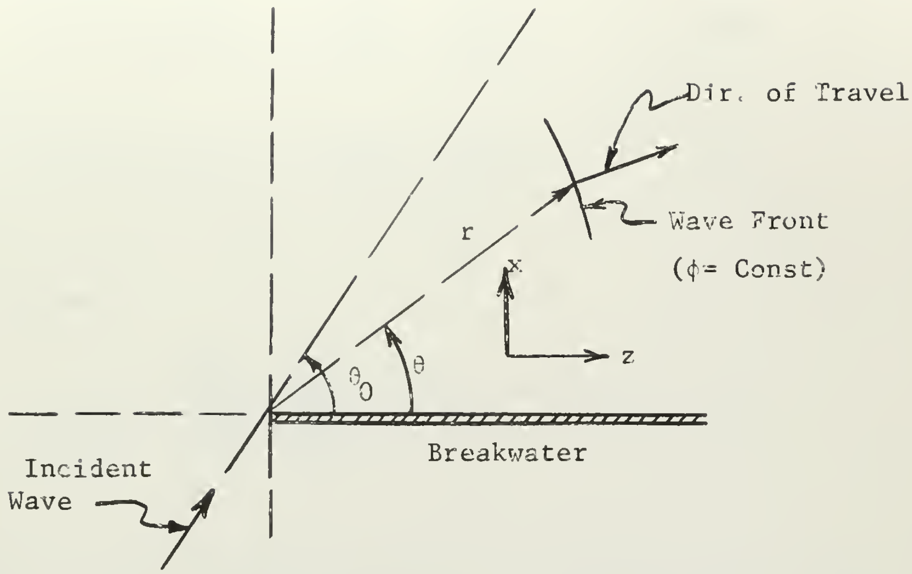
a) Coefficient of Diffraction, K'

Except for evaluation of Fresnel Integrals, equations (23) through (42) can be solved on a computer in essentially their present form. In the programs developed, a completely random orientation of breakwaters and incident waves was desired. This and a desire for reasonable program efficiency and memory size cause substantial bookkeeping problems but create no need for any new methods or procedures.

As shown in Appendix A, a series solution was developed for the evaluation of the Fresnal Integrals, This was used for sigma values less than 3.0. For larger values, sine/cosine approximations given by Sparrow [23] were used. If used with sigma values greater than approximately 3.0, the developed series solution requires greater precision than normally used on most computers. Double precision could be used but tests indicated that satisfactory evaluation, good to within 0.1 percent of the Integral, could be achieved by using the two methods. A third approximation formula is given by Jahnke and Ende [22] which is suitable for evaluation of values in the intermediate range between the series solution and the sine/cosine approximations but this was not felt necessary.

b) Direction of Travel of Wave Crest

As noted earlier in equation (43), the phase, ϕ, of the diffracted wave is given by $\phi = \tan^{-1}(B/A)$



Wave Front Nomenclature

Figure 7

The equation of the wave crest is given by

$$\phi\ (z,x) = \text{Const}$$

The normal to the wave crest is a vector which is given by

$$\text{grad}\ (\phi) = \frac{\partial \phi}{\partial x}\ \vec{i} + \frac{\partial \phi}{\partial x}\ \vec{k}$$

in which $\vec{i}$ and $\vec{k}$ were the unit vectors in the x and z directions respectively. Therefore, representing the slope of the normal as $[\frac{dx}{dz}]_{\perp}$ we have

$$[\frac{dx}{dz}]_{\perp} = \frac{\partial \phi / \partial x}{\partial \phi / \partial z} \qquad (44)$$

The partial derivatives in (44) can be expressed in terms of A and B. Using (43) this gives

$$\frac{\partial \phi}{\partial z} = \frac{1}{1+(B/A)^2} \left[\frac{1}{A}\frac{\partial B}{\partial z} - \frac{B}{A^2}\frac{\partial A}{\partial z}\right] \qquad (45)$$

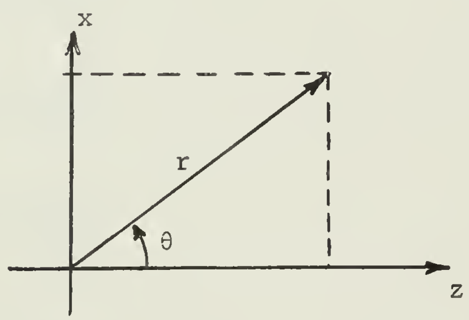
$$\frac{\partial \phi}{\partial x} = \frac{1}{1+(B/A)^2} \left[\frac{1}{A}\frac{\partial B}{\partial x} - \frac{B}{A^2}\frac{\partial A}{\partial x}\right] \qquad (46)$$

Hence (44) becomes

$$\left[\frac{\partial X}{\partial Z}\right]_{\perp} = \frac{A\dfrac{\partial B}{\partial X} - B\dfrac{\partial A}{\partial X}}{A\dfrac{\partial B}{\partial Z} - B\dfrac{\partial A}{\partial Z}} \qquad (47)$$

Equation 47 is our desired solution for the direction of wave travel.

Given A and B from equations (39) or (40), we must now evaluate $\partial A/\partial X$, $\partial B/\partial X$, $\partial A/\partial Z$ and $\partial B/\partial Z$. For these we will need the derivatives $\partial r/\partial X$, $\partial r/\partial Z$, $\partial \theta/\partial X$, and $\partial \theta/\partial Z$. For the breakwater along the +Z axis,



$$x = r \sin \theta$$

$$z = r \cos \theta$$

$$\tan \theta = \frac{x}{z}$$

Therefore,

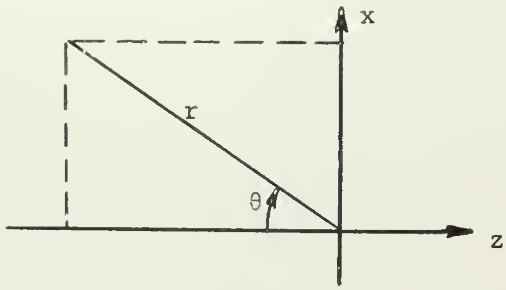
$$\frac{\partial r}{\partial x} = \sin \theta$$

$$\frac{\partial r}{\partial z} = \cos \theta$$

$$\frac{\partial \theta}{\partial x} = \frac{\cos\,\theta}{r}$$

$$\frac{\partial \theta}{\partial z} = -\,\frac{\sin\,\theta}{r}$$

For the breakwater along the -z axis



$$x = r\,\sin\,\theta$$

$$z = -r\,\cos\,\theta$$

$$\tan\,\theta = -\,\frac{x}{z}$$

Therefore,

$$\frac{\partial r}{\partial x} = \sin\,\theta$$

$$\frac{\partial r}{\partial z} = -\cos\,\theta$$

$$\frac{\partial \theta}{\partial x} = \frac{\cos\,\theta}{r}$$

$$\frac{\partial \theta}{\partial z} = \frac{\sin\,\theta}{r}$$

If we consider X = f(r,θ), the chain rule gives

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial B}{\partial \theta}\frac{\partial \theta}{\partial x}$$

which becomes

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial r}\,\sin\,\theta + \frac{\partial B}{\partial \theta}\,\frac{\cos\,\theta}{r}$$

Similarly, one gets

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial r} \sin \theta + \frac{\partial A}{\partial \theta} \frac{\cos \theta}{r}$$

Using the same techniques we can evaluate $\frac{\partial A}{\partial z}$ and $\frac{\partial B}{\partial z}$; but in this case, results differ with the breakwater orientation. If the breakwater is along the +z axis,

$$\frac{\partial B}{\partial z} = \frac{\partial B}{\partial r} \cos \theta - \frac{\partial B}{\partial \theta} \frac{\sin \theta}{r}$$

$$\frac{\partial A}{\partial z} = \frac{\partial A}{\partial r} \cos \theta - \frac{\partial A}{\partial \theta} \frac{\sin \theta}{r}$$

while if it is along the -z axis

$$\frac{\partial B}{\partial z} = - \frac{\partial B}{\partial r} \cos \theta + \frac{\partial B}{\partial \theta} \frac{\sin \theta}{r}$$

$$\frac{\partial A}{\partial z} = - \frac{\partial A}{\partial r} \cos \theta + \frac{\partial A}{\partial \theta} \frac{\sin \theta}{r}$$

Still using the notation of Fig. 7 the following convenient notations are used:

$$k = 2\pi/CT$$

$$FM = kr \cos (\theta-\theta_0)$$

$$\frac{\partial FM}{\partial r} = k \cos (\theta-\theta_0)$$

$$\frac{\partial FM}{\partial \theta} = -kr \sin (\theta-\theta_0)$$

$$FP = kr \cos (\theta+\theta_0)$$

$$\frac{\partial FP}{\partial r} = k \cos (\theta+\theta_0)$$

$$\frac{\partial FP}{\partial \theta} = - kr \sin (\theta+\theta_0)$$

Introducing this notation into equations (39) and (41), in region S, one has

$$A = U1 \cos(FM) + U2 \cos(FP) + W1 \sin(FM) + W2 \sin(FP) \tag{48}$$

Taking derivatives

$$\frac{\partial A}{\partial r} = \frac{\partial U1}{\partial r} \cos(FM) - U1 \sin(FM) \frac{\partial FM}{\partial r} + \frac{\partial U2}{\partial r} \cos(FP) - U2 \sin(FP) \frac{\partial FP}{\partial r}$$

$$+ \frac{\partial W1}{\partial r} \sin(FM) + W1 \cos(FM) \frac{\partial FM}{\partial r} + \frac{\partial W2}{\partial r} \sin(FP) + W2 \cos(FP) \frac{\partial FP}{\partial r}$$

$$\frac{\partial A}{\partial \theta} = \frac{\partial U1}{\partial \theta} \cos(FM) - U1 \sin(FM) \frac{\partial FM}{\partial \theta} + \frac{\partial U2}{\partial \theta} \cos(FP) - U2 \sin(FP) \frac{\partial FP}{\partial \theta}$$

$$+ \frac{\partial W1}{\partial \theta} \sin(FM) + W1 \cos(FM) \frac{\partial FM}{\partial r} + \frac{\partial W2}{\partial \theta} \sin(FP) + W2 \cos(FP) \frac{\partial FP}{\partial \theta}$$

Similarly,

$$B = W1 \cos(FM) + W2 \cos(FP) - U1 \sin(FM) - U2 \sin(FP) \tag{49}$$

$$\frac{\partial B}{\partial r} = \frac{\partial W1}{\partial r} \cos(FM) - W1 \sin(FM) \frac{\partial FM}{\partial r} + \frac{\partial W2}{\partial r} \cos(FP) - W2 \sin(FP) \frac{\partial FP}{\partial r}$$

$$- \frac{\partial U1}{\partial r} \sin(FM) - U1 \cos(FM) \frac{\partial FM}{\partial r} - \frac{\partial U2}{\partial r} \sin(FP) - U2 \cos(FP) \frac{\partial FP}{\partial r}$$

$$\frac{\partial B}{\partial \theta} = \frac{\partial W1}{\partial \theta} \cos(FM) - W1 \sin(FM) \frac{\partial FM}{\partial \theta} + \frac{\partial W2}{\partial \theta} \cos(FP) - W2 \sin(FP) \frac{\partial FP}{\partial \theta}$$

$$- \frac{\partial U1}{\partial \theta} \sin(FM) - U1 \cos(FM) \frac{\partial FM}{\partial \theta} - \frac{\partial U2}{\partial \theta} \sin(FP) - U2 \cos(FP) \frac{\partial FP}{\partial \theta}$$

Similar equations for A, B, and their derivatives apply in region Q or at the breakwater gap and may be derived from equations (40) and (41), or (44) and (41), respectively. These equations are not detailed

in this report in view of the fact that they offer nothing new in the way of knowledge and are simply exercises in bookkeeping.

For evaluation of these last groups of derivatives, one must determine U1, U2, W1, W2 and their derivatives. Consider first the evaluation of U1 and W1. We recall from (31), (32), and (35)

$$U1 = \frac{1}{2}(1-S-C)$$

$$W1 = \frac{1}{2}(S-C)$$

Taking derivatives

$$\frac{\partial U1}{\partial r} = -\frac{1}{2}\left(\frac{\partial S}{\partial r} + \frac{\partial C}{\partial r}\right)$$

$$\frac{\partial W1}{\partial r} = \frac{1}{2}\left(\frac{\partial S}{\partial r} - \frac{\partial C}{\partial r}\right)$$

$$\frac{\partial U1}{\partial \theta} = -\frac{1}{2}\left(\frac{\partial S}{\partial \theta} + \frac{\partial C}{\partial \theta}\right)$$

$$\frac{\partial W1}{\partial \theta} = \frac{1}{2}\left(\frac{\partial S}{\partial \theta} - \frac{\partial C}{\partial \theta}\right)$$

To evaluate derivatives of S and C it is convenient to set

$$t = \frac{\pi\sigma^2}{2}$$

which gives

$$\frac{\partial t}{\partial r} = \pi\sigma\frac{\partial \sigma}{\partial r} \tag{50}$$

Equation (29) therefore gives

$$S(t_1) = \int_0^{t_1} \frac{\sin t}{\sqrt{2\pi t}} \, dt$$

$$\frac{dS}{dr} = \frac{dS}{dt} \frac{dt}{dr} = \frac{dt}{dr} \left(\frac{\sin t}{\sqrt{2\pi t}}\right)$$

substituting (50)

$$\frac{dS}{dr} = \sin\left(\frac{\pi\sigma^2}{2}\right) \frac{d\sigma}{dr} \qquad \left(\frac{dS}{dr} \text{ keeps sign of } \sigma\right)$$

Also from equation (28) we derive

$$C(t_1) = \int_0^{t_1} \frac{\cos t}{\sqrt{2\pi t}} \, dt$$

$$\frac{dC}{dr} = \frac{dC}{dt} \frac{dt}{dr} = \frac{dt}{dr} \left(\frac{\cos t}{\sqrt{2\pi t}}\right)$$

$$\frac{dC}{dr} = \cos\left(\frac{\pi\sigma^2}{2}\right) \frac{d\sigma}{dr} \qquad \left(\frac{dC}{dr} \text{ keeps sign of } \sigma\right)$$

Using the same techniques we can evaluate θ derivatives of S and C and get

$$\frac{dS}{d\theta} = \sin\left(\frac{\pi\sigma^2}{2}\right) \frac{d\sigma}{d\theta} \qquad \left(\frac{dS}{d\theta} \text{ keeps sign of } \sigma\right)$$

$$\frac{dC}{d\theta} = \cos\left(\frac{\pi\sigma^2}{2}\right) \frac{d\sigma}{d\theta} \qquad \left(\frac{dC}{d\theta} \text{ keeps sign of } \sigma\right)$$

From (23) we can derive

$$\sigma = 2\sqrt{\frac{kr}{\pi}}\ \sin\frac{1}{2}(\theta-\theta_0)$$

$$\frac{\partial\sigma}{\partial r} = \sqrt{\frac{k}{\pi r}}\ \sin\frac{1}{2}(\theta-\theta_0) = \frac{\sigma}{2r}$$

$$\frac{\partial\sigma}{\partial\theta} = \sqrt{\frac{kr}{\pi}}\ \cos\frac{1}{2}(\theta-\theta_0)$$

We evaluate U2, W2 and their derivatives similarly, using σ' and its derivatives, i.e.

$$\sigma' = -2\sqrt{\frac{kr}{\pi}}\ \sin\frac{1}{2}(\theta+\theta_0)$$

$$\frac{\partial\sigma'}{\partial r} = -\sqrt{\frac{k}{\pi r}}\ \sin\frac{1}{2}(\theta+\theta_0) = \frac{\sigma'}{2r}$$

$$\frac{\partial\sigma'}{\partial\theta} = -\sqrt{\frac{kr}{\pi}}\ \cos\frac{1}{2}(\theta+\theta_0)$$

Therefore, given values of r, θ, θ_0, and k, we can back substitute through the preceeding series of equations to ultimately evaluate the direction of wave travel as given by equation (47).

c) Alternate Methods Considered

The two methods used in diffraction analysis by engineers have been the Penney-Price and Morse Rubenstein methods. When evaluated by Carr and Stelzride [25], both were found to have advantages and disadvantages but the Penney-Price method was found to be the better. Ever since, the Penney-Price has been used almost exclusively except in the

evaluation of breakwater gaps where the more exact theoretical solutions of Morse Rubenstein have often been used. Principally due to the fact that more testing had been accomplished and much more information was available in publications on the Penney-Price methods, this approach was used in the analysis.

Once committed to this method, when it was later decided to add solutions of the breakwater gap problem to the program package, the Penney-Price solutions using the imaginary breakwater gap developed by Johnson were used in the interests of simplicity. Although not an exact solution when applied in the vicinity of the breakwater wings, in the hands of an experienced engineer these areas should cause no real problems.

Many simplifications which omit negligible terms in various regions of a diffraction zone could be made. Due to the unknown effect which these approximations would have in the determination of the direction of wave travel, an exact solution was used throughout. The actual computer time saved by making approximations would be very small and was not felt to be worth the chance of having to rewrite programs from approximate to more exact solutions if the approximations did not work. Working from an exact solution however, approximations could be added at a later time and results readily verified against the original solution.

E. Tests applied

To test the accuracy of these programs, they were used to calculate diffraction coefficients and directions of wave travel at many random points using various breakwater coordinates and various directions of travel for the incident wave. Results were then checked against graphical solutions which have been published.

Problems were encountered in this verification however, due to the fact that virtually every published refraction diagram was based on approximations of one kind or another (usually not mentioned in the write up) which had a substantial effect on the quality of the plot. In fact, no two published diffraction diagrams from different authors appeared to agreee with each other as a result of these various approximations. The program results did not completely agree with any published diffraction diagram. However, in the calculation of the value for a coefficient of diffraction, good correlation was found between the program and the diffraction chart in Wiegel (8, p.183). Also, in the calculation of the direction of wave front travel, good correlation was found with the diffraction chart published by the Beach Erosion Board (9, p.38). Table 2 compares program results with diffraction coefficients interpolated from Wiegel and with directions of wave travel interpolated from the Beach Erosion Board. It should be remembered that in both cases values interpolated off the charts are no better than the accuracy with which the chart was drawn or one's ability to interpolate off them. In addition, analysis of the basic equations, especially in the region around and beyond the breakwater tip, where the most discrepancies are found between published charts, indicates that the results achieved are in fact reasonable and in good agreement with the complete theoretical solution.

F. Limitations of Numerical Diffraction Programs

In that these programs are built on the same theoretical basis as the past plots, they suffer the same limitations. No new theories or improved results can be claimed over a completely accurate plot. The program's chief asset is its ability to produce results in a small fraction of the time required by graphical methods.

Table 2.

Diffraction Analysis

θ_0	θ	r (grids)	Test K'	Wiegel[1] K'	Test Dir. of Trav.	Published Dir. of Trav.[2]
90°	45.00	7.32	0.107	0.110	43.89	45.$^{(-)3}$
90	63.43°	2.24	0.261	0.26	61.38	62.
90	90.00	4.00	0.529	0.53	83.62	82.
90	90.00	8.00	0.520	0.52	85.46	85.
90	116.56	4.44	1.108	1.1+	92.46	92.
90	122.00	9.44	1.060	1.07	91.81	90.+
90	135.00	2.83	0.937	0.9[3]	87.1	86.
90	104.04	8.25	1.097	1.0	91.99	92.
45	77.56	4.44	1.104	1.0	37.37	Not Avail.
45	45.00	3.00	0.549	0.55	37.60	" "
45	18.43	4.44	0.228	0.3	17.67	" "
135	165.96	5.83	0.951	1.0	133.31	" "
135	135.00	8.00	0.537	0.53	140.75	" "
135	45.00	8.00	0.086	0.06	45.00	" "
GAP (90°)	63.45	6.32	0.239	0.34[4]	77.80	75.[4]
GAP	90.00	5.00	0.849	0.85[4]	90.00	90.[4]
GAP	104.04	2.06	0.888	0.8[4]	94.29	92.[4]

1. Wiegel [8] p. 184 unless otherwise noted.

2. Beach Erosion Board [9] p. 38 unless other noted

3. Wiegel [8] p. 183

4. Wiegel [8] p. 188

Its output is still no better than its input data. Study of complete wave spectra are still needed for thorough analysis of an area. The program should however make this analysis easier than it has been in the past.

There are still differences between theoretical and experimental results as shown by Putnam and Arthur [20], Blue and Johnson [26], and Carr and Stelzride [25], which were mentioned in Section III B. Although the agreement between theory and experiment is satisfactory, theory does overestimate wave heights by up to 10 percent in the zone beyond a breakwater tip. This is probably due to a significant loss in wave energy during the process of reflections, (Terry [27]). All calculations are based on ideal breakwaters with 100 percent reflection. In practice incident waves are only partly reflected and are partly destroyed by turbulence (Penney and Price [3]).

A basic assumption is that the wave height is small compared to its length and that the wave profile is nearly sinusoidal. Although the experiments mentioned above indicated that the results are not seriously altered by waves of moderate height, design for solitary waves likely to be generated during severe storm conditions could be susceptible to error.

Gap solutions have only been experimentally verified by Blue and Johnson [26] with a wave angle of approach less than about 60 degrees; however, they are the only solutions available so they are actually used in practice over the entire 90 degree range.

For short breakwaters of the ribbon type, with diffraction about both ends, the program solutions would not be valid in a region of diffraction effects from both ends. In practice however, the breakwaters are usually of sufficient length to achieve good results by diffracting each end independently.

IV. PROGRAM SUMMARY AND ENGINEERING FEATURES

The program developed was written so as to facilitate its use by engineers in the analysis of coastal areas. The following summaries give highlights of their operation. Details of the main program and all subroutines are explained in the appendix.

(1) Main Program

The purpose of the main program is to read data and commands, set variables, and call on the various subroutines to take appropriate action when necessary. The main program is written so as to read problem-oriented language input. In this way, the operator uses the same language in describing input data and tasks to be performed as he uses in normal every day work. When appropriate, a written output of data is provided to assure a complete record of the calculations performed.

An attempt was made to provide most of the commands that an engineer would have occasion to use in an analysis of a coastal area. Although this program was, of necessity, written for batch processing, it is well suited to console input and random interrogation by an operator following the progress of a solution.

Since much of the refraction/diffraction analysis process is still left to engineering judgment, the flexibility was provided for the engineer to use his judgment whenever it is deemed necessary. A minimum of information is set by the program, it depends on an engineer for important variables.

(2) Refraction Subroutine

This subroutine traces wave rays across an array of water depths until the ray goes onto a beach, goes off an edge of the array, or enters a breakwater zone. During this trace, all information concerning the wave ray which could be of use to an engineer is output, including location, water depth, speed, direction of travel, shoaling coefficient, and coefficient of refraction.

Due to the fact that in many cases refraction analysis is made in breakwater areas, this program stops any wave traces upon crossing a breakwater or upon entering a breakwater diffraction zone. At these points of intersection, all necessary wave ray information is determined and output for use by the operator in further analysis.

(3) Diffraction Subroutine

This subroutine calculates the coefficient of diffraction and direction of wave front travel in a diffraction zone. This may be a diffraction zone at the end of a semi-infinite breakwater, or a diffraction zone at a gap between two breakwaters. Given the location of one or two breakwaters and necessary data on the incident wave, this progran analyzes the diffraction pattern at any specified point and outputs the coefficient of diffraction and the wave front direction of travel.

(4) Shoreline Tracing

This subroutine will find and trace shorelines across the array of water depths. Although this feature is most useful if the program is converted to some sort of a graphical output device, it can also be helpful when working in an area where tidal fluctuations can have a

noticeable effect on the shoreline location. Once a depth array is input to the program, presumably based on a "mean-sea level", tidal changes can be simulated through the input of water level changes; if these changes affect the shoreline, this program will determine its new location.

(5) General Features

To facilitate maximum adoption of this system by any interested users, it was written in the FORTRAN IV programming language. Although written for the IBM 360 computer, it can readily be adapted to most other hardware simply by changing the alphameric character constants in MAIN and by adopting the read/write statements to available devices.

The program is now written for any water depth array size up to 50 by 40 grid units. This can be changed up or down as desired, simply by changing the DIMENSION cards. No other changes are needed.

The entire system at present requires about 10,000 words of memory on an IBM 360 - Model 40 computer. However for smaller cores, it is possible to use the program in portions. For example, during refraction calculations, subroutines, RSSHOR which locates the shoreline, and RSDIFF which performs diffraction calculations, are not needed and minimum sized dummy subroutines may be inserted for them. During diffraction calculations, only RSDIFF and MAIN are needed. For tracing a shoreline, only MAIN and RSSHOR are needed. Although this method of operation would cut down on some of the operator flexibility, it can facilitate program use on hardware too small to accept the entire package and it could enable an operator to trade off unnecessary program for an increased array size.

V. POSSIBLE SYSTEM EXPANSION AND IMPROVEMENTS

The programs developed show some of the possible applications of a computer by an engineer in the study of coastal areas. While writing these programs it became apparent that many additions and improvements could be made to them which would still further facilitate engineering analysis. The following are some of these possible expansions. These explanations are not intended to be complete analyses but are only to point out possible areas of further study.

(1) Wave Front Tracing

In many studies, plots of wave fronts can be very useful in graphically depicting a situation. The present program, by tracing wave rays, will not give any wave front data other than the fact that the wave fronts are orthogonal to the ray and simulated fronts could possibly be added to the rays by an experienced draftsman or engineer. It would be possible however to expand the present program to output wave front coordinates. The present ray trace program is based on increments of equal distance between points. This could be changed to increments of equal time. As the iteration process, which is needed to locate succeeding points, take place, an average speed would be calculated and the interval between points could be based on this average wave speed and the given periods of equal time. If all wave rays were started along a wave crest, (this ability to start multiple rays could also be added to the program) each successive coordinate point output by each ray would be a successive crest position. This feature would also have the beneficial effect of reducing the increment interval as the ray moves into shallower depths.

(2) Graphical output

The refraction/diffraction problem is ideally suited for a graphical output. The combination of graphical output and console input by an engineer would be an extremely powerful tool in the analysis of any coastal area. Early versions of the program were run on an IBM 1620 connected to a Gerber Plotter. Unfortunately, the program was not perfected prior to removal of this facility from M.I.T.

Although not used by the author, perhaps the most interesting and promising output would be accomplished on an oscilloscope. Through a combination of console and light pen input to the program, an engineer could rapidly analyse diffraction and refraction problems. With a minimum of effort he could investigate far more possible solutions than would be practical by any other method.

(3) Breakers and Surf

At present, most predictions of breaker and surf locations along a shoaling beach are done empirically. These locations can be very important in the analysis of a beach due to the fact that refraction does not take place beyond the breaker zone and direction of travel of the breaker is very important in determining littoral currents. If breaking or surf occurs on a reef at some distance from a shoreline, especially at different tidal elevations, this can be very important. For some studies therefore, it may be desirable to add to the program a means of predicting the breaker location. This could be done by determining a desired breaking criteria and adding it as another check item to the refraction calculations.

(4) Refraction by Currents

In some areas, especially at estuaries, one may have to allow for refraction caused by currents. In the past, using graphical methods, it was impractical to account for local currents in refraction studies and they were either neglected (Dunham [13]) or accounted for by engineering judgement. A theory for the refraction effects of currents is well established and has been published in some detail by Johnson [10]. Due to the possible occurrence of currents in coastal areas resulting from long shore currents, currents caused by topography, flow from regions of high refraction to areas of low refraction, eddy currents, and rip currents, it may be advantageous to have an ability to account for their effects. This would be a desirable addition to the present program.

(5) Output Summaries

A suitable means of summarizing output is needed. Various methods of portraying refraction for a location, showing the effects of period, and direction of wave travel have been used in the past by Johnson [10], and the Beach Erosion Board [9]. As the volume of data increases however, most of these methods become confusing. It would appear that this is an area requiring considerable common sense on the part of the engineer to assure that a detailed study with noteworthy results is not lost or obscured by a poor presentation. It would be most desirable, of course, to have a suitable data presentation come directly from the computer.

(6) Approximations for Diffraction Process

As noted earlier, diffraction theory, when applied to water waves, generally gives conservative results. It was also found that in many instances, approximations of the complete theoretical solution brought results into closer agreement with the tests than did the complete solutions, (Blue and Johnson [26]). The Beach Erosion Board [9] points out that because of non-uniform validity of various orders of wave theories, the best approximation is not necessarily the highest order. LaCombe [30] proposed approximate solutions to the diffraction problem which may offer some avenues of approach. It may therefore be desirable to adjust the present program to allow for various approximations. These approximations can be compared with the complete solution and can be checked against experimental data for an empirical comparison.

(7) Command structure

To the maximum extent possible, the program was designed to facilitate expansion and improvement. Additional commands may be added to MAIN simply by adding their appropriate alphameric characters to the listing and adding a block of appropriate instructions to the program. The existing commands could be improved through the addition of allowable abbreviations and shorter names. Input is well suited to free format and this would be a most desirable feature, especially if console input is to be used.

(8) Multiple Waves

In many instances, different period waves and waves travelling from different directions will arrive at a given point at the same

time. There are methods for determining the resultant of multiple waves as shown by Eagleson, et al [7]. This is not a simple addition problem and, in view of its complexity and common occurrence, it would be a desirable feature which a design engineer could use.

(9) Combined Refraction and Diffraction

All diffraction theory is based on waves of constant speed. In practice, breakwaters are usually built in areas where shoaling is an important part of the problem and the wave speed variation is important. To date, no combined theory of refraction and diffraction has been developed. According to Bretschneider [12], and many other authors, the only satisfactory means of studying combined refraction/diffraction problems is to refract a wave to a breakwater zone, diffract the wave using an average wave speed over an arbitrary number of wave lengths, then refract the wave again to the shoreline. Products of refraction coefficients and diffraction coefficients are used to determine final wave heights.

Some method of calculating combined refraction/diffraction problems would remove these arbitrary evaluations required at present.

VI. CONCLUSIONS

The basic purpose of this study was to develop a computer system for use by engineers in the analysis of wave behavior in coastal areas.

The refraction program developed was based on 3-dimensional continuous parabolic interpolation of a depth array. This procedure gives continuous curves through any series of data points. Adapted to 3-dimensions and used for refraction analysis, it appears to offer excellent results.

The diffraction program is based on a form of the basic Penney-Price water wave diffraction methods. The system developed calculates diffraction coefficients and directions of wave travel which compare closely with manual/graphical plots.

The entire program package offers a means of analyzing a coastal area in whatever depth the engineer desires in but a small fraction of the time required by older methods. Hopefully, this will facilitate more thorough design than has been economically feasible in the past.

More testing of the programs is required; hopefully against situations where natural conditions are known and where output could be compared to actual phenomena.

Refraction/diffraction analysis is ideally suited for graphical plots of the output. With the present program system as a basis, output should be adapted to some sort of plotter or oscilloscope where solutions in progress could be watched and evaluated by an engineer. With console input, an engineer watching plots of his solutions would be able to try various design schemes for a facility and thoroughly evaluate their effects in but a few minutes.

Although improvements and further testing are needed, the system developed appears to offer considerable promose for better analysis of coastal refraction/diffraction problems than has been possible in the past.

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VIII. APPENDICES

A. Fresnel Integral Calculations

B. Program Structure

Each program includes:
- a. Listing of Variables
- b. Summary of Operations
- c. Macro-Flow Chart
- d. Program Listing

1. MAIN - Program to read in all commands and data and take or call for appropriate action.
2. RSRFTN - Subroutine to trace a wave ray through shoaling water.
3. RSINTP - Subroutine that calculates depth and bottom derivatives at any point.
4. RSBETA - Subroutine which calculates ray separation factor, coefficient of refraction and shoaling factor.
5. RSCAFK - Subroutine which calculates wave velocity and curvature at any point.
6. RSDIFF - Subroutine which calculates coefficient of diffraction and wave direction of travel in a diffraction zone.
7. RSSHOR - Subroutine which locates and traces shoreline.

C. Format for Command/Data Input

D. Refraction Test on Theoretical Beach

E. Sample Run

APPENDIX A

EVALUATION OF FRESNEL INTEGRALS

A Fresnel Integral is defined by:

$$\int_0^{\sigma} e^{-\pi i\, u^2/2}\, du \qquad (1)$$

Letting $t = \pi\, u^2/2$ and using $t_1 = \pi\sigma^2/2$ as a limit we can set

$$(1) = \int_0^{t_1 = \frac{\pi\sigma^2}{2}} \frac{e^{-it}}{\sqrt{2\pi t}}\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \left(\int_0^t \frac{1}{\sqrt{t}} \cos t\, dt - i \int_0^t \frac{1}{\sqrt{t}} \sin t\, dt\right)$$

$$= C - iS$$

Evaluation of C

$$C = \frac{1}{\sqrt{2\pi}} \int_0^{t_1} \frac{1}{\sqrt{t}} \cos t\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{t_1} \frac{1}{\sqrt{t}} \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \text{---}\right) dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{t_1} \left(t^{-1/2} - \frac{t^{3/2}}{2!} + \frac{t^{7/2}}{4!} - \frac{t^{11/2}}{6!} + \text{---}\right) dt$$

$$= \frac{1}{\sqrt{2\pi}} \left(2t^{1/2} - \frac{2t^{5/2}}{5 \cdot 2!} + \frac{2}{9} \frac{t^{9/2}}{4!} - \frac{2}{13} \frac{t^{13/2}}{6!} + \text{---}\right)$$

using

$$p = t^{1/2} = \sqrt{\frac{\pi}{2}}\ \sigma \quad \text{we get}$$

$$C = \frac{2}{2\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ p^{4(n-1)+1}}{(4(n-1)+1)(2(n-1))!}$$

Similarly:

$$S = \frac{1}{\sqrt{2\pi}} \int_0^{t_1} \frac{1}{\sqrt{t}} \sin t \, dt$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{2}{3}\ t^{3/2} - \frac{t^{7/2}}{7\cdot 3!} + \frac{t^{11/2}}{11\cdot 5!} - \frac{t^{15/2}}{15\cdot 7!} + \text{---}\right)$$

$$S = \sqrt{\frac{2}{\pi}} \sum_{n=1}^{\infty} \frac{p^{4(n-1)+3}\ (-1)^{n+1}}{(4(n-1)+3)(2n-1)!}$$

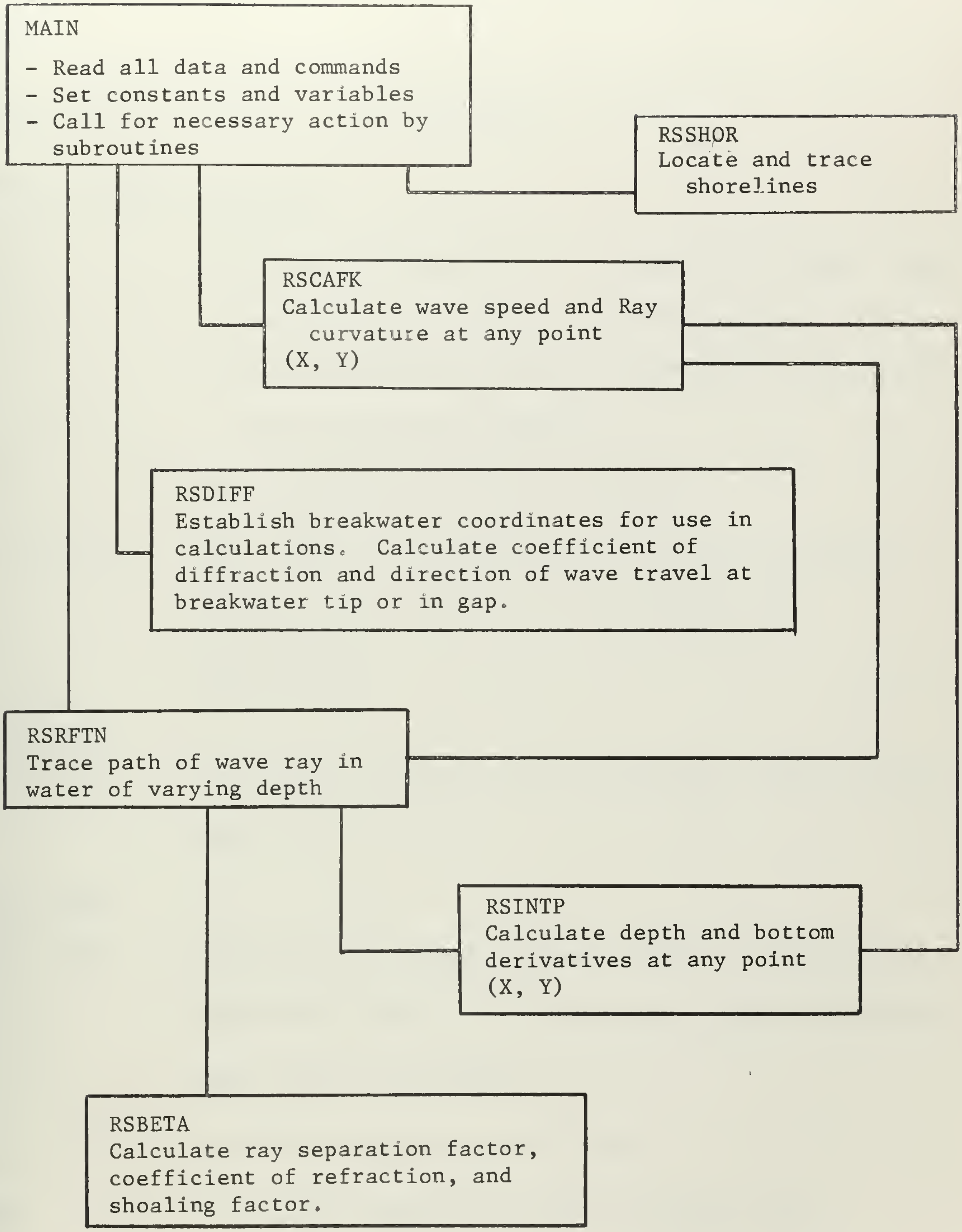
The following formulae as given by Sparrow [23] can be used to approximate C and S. For large values of t,

$$C = 0.5 + \left(\frac{1}{\pi\sigma}\right) \sin\left(\frac{\pi}{2}\left(\sigma^2 - \frac{2}{\pi^2\sigma^2}\right)\right)$$

$$S = 0.5 - \left(\frac{1}{\pi\sigma}\right) \cos\left(\frac{\pi}{2}\left(\sigma^2 - \frac{2}{\pi^2\sigma^2}\right)\right)$$

APPENDIX B

PROGRAM STRUCTURE



1. MAIN PROGRAM

Main program to read data cards and take or call for appropriate action.

PROGRAM NAME: Referred to as MAIN in text.

Variables stored in Common

A . . . . 1) Angle in radians from +X direction to wave ray. Angle is measured in positive (counterclockwise) direction.
2) In diffraction, the angle of approach of the incident wave ray at the breakwater.

AD. . . . . . . A in degrees; used for input/output.

BN, B1, B2 . . Beta, wave ray separation factor, at point preceeding (X,Y), at (X,Y) and at point succeeding (X,Y), respectively.

BXAS, BYAS,
BXBS, BYBS. . . Coordinates of ends of first breakwater, (A-B) on DMAT grid.

BXCS, BYCS,
BXDS, BYDS. . . Coordinates of second breakwater (C-D). If GAP is specified it must lie between B and C ends of breakwater.

C . . . . Wave velocity in ft/sec.

CN . . . . . . G*T/(2*PI); deep water wave speed.

CPPK . . . . . C *T/(4*PI); referred to as CK" or K"C in text.

DCDX, DCDY. . . $\partial C/\partial X$, $\partial C/\partial Y$ in text; partial derivatives of wave speed to X and Y, respectively.

DDDC . . . . . dd/dc in text; derivative of depth to wave speed.

DMAT . . . . . Matrix of water depths in feet. Positive indicates water depth; negative is above shoreline.

DXY . . . . . . Depth at (X,Y). ("d" in text).

DIX, DIY . . . $\partial d/\partial X$, and $\partial d/\partial Y$ in text; slope of bottom at (X,Y).

D2X,D2Y,D2XY . $\partial^2 d/\partial X^2$, $\partial^2 d/\partial Y^2$, $\partial^2 d/\partial X \partial Y$ in text; partial second derivatives of bottom depth at (X,Y).

FK. . . . . . . Wave ray curvature at (X,Y). Equals inverse of radius of curvature.

G . . . . Acceleration due to gravity; 32.2 ft/sec.

GRID . . . . . Size of each DMAT grid unit in feet.

IXN,IYN . . . . Maximum matrix dimensions being used in text. These can be less than or equal to dimension statement and are used in lieu of changing DIMENSION statement. Can use any DIMENSION statement which will fit in memory with rest of programs.

NBW . . . . . . Index for counting breakwaters; NBW=1, if none; 2 if one breakwater; 3 if two breakwaters (no gap); 4 if two breakwaters and gap between them.

PI. 3.1415927

PK . . . . . . T/(4*PI); referred to as K' in text.

PPK . . . . . . (2*PI)/(G*T); referred to as K" in text.

S. . . . . Incremental distance, in grid units, between successive calculation points along a wave ray.

T. . . . . Wave period in seconds.

WLD . . . . . . Length of diffraction zone beyond ends of breakwaters; either in feet or number of wave lengths, depending on input.

X,Y. . . . . . . Coordinates of position within DMAT which are being studied.

Variables used by MAIN

DMMY . . . . . . Dummy variable, no significance.

DX,DY,DM . . . . Used in establishing analytical depth matrices; DM is the depth at DMAT (IX1, IY1); DX is the increase in depth per X grid unit; DY is the increase in depth per Y grid unit.

I,J . . . . . . "Do Loop" indexes.

IX,IY . . . . . Grid coordinates on DMAT (IX,IY) array.

IX1,IX2,

IY1,IY2 . . . . Limits of operations being performed on DMAT.

IWD (--) . . . . Listing of decimal equivalents of "A" format characters for IBM 360.

IWD1,IWD2. . . . "A" format characters read off command portion of input.

IXP1,IYP1. . . . Convenient totals IX + 1; IY + 1.

LL,LL2 . . . . . . Index used to check through IWD(LL) array.

LLS . . . . Value of LL read on last command. Needed if present command is a blank which indicates repeat of last command.

NN . . . . Index used to indicate program status when calling RSCAFK: NN=1, calc. depth and only C; NN=2, depth is known, calc. C and FK.

NWL. . . . . Index to indicate means of measuring length of diffraction zone. (See WLD).
NWL=1 if measured in wave lengths.
NWL=2 if measured in feet.

N1,N2,F1,F4 . . Fixed and floating point data values read off command card. These will be set to actual variables after command is decoded.

Summary of Program

The purpose of MAIN is to read all command and data cards, then, either within itself or by calling subroutines, take whatever action is required. Its principal functions are bookkeeping, establishing variables for use by subroutines, and calling subroutines to take whatever action has been called for by a command. In some instances, variables are predetermined; in some, a value is assumed unless changed by the operator; and in others, operator input is essential. The proper format of all commands is shown in Appendix C.

Command/Data Input

See Appendix C for proper format. Underscore indicates Alphameric characters which are used in decoding command and which must occur in columns 1 and 10. Abbreviations which do not change underscored characters are allowed.

Required input for Refraction Calculations:

Either an analytical or topographic array or a combination of the two must be established.

ANALYTICAL MATRIX	IX1	IX2	DX	DM
	IY1	IY2	DY	

Two cards are required. A plane surface is established which extends from IX1 to IX2 with an increase in depth of DX per X grid unit; and extends from IY1 to IY2 with increases of DY per Y grid unit. DM is the depth of (IX1, IY1). All depths are in feet.

LIMITS OF TOPO	IX1	IX2
	IY1	IY2

Followed by:

TOPO IN FEET FOLLOWS

(or)

TOPO IN FATH FOLLOWS

Followed by:

Data in 10 F6.4 Format

This is a series of commands used in reading natural beaches into array which extend from IX1 to IX2 and IY1 to IY2. If topography is in fathoms, the program immediately converts it to feet.

Data is read one X row at a time, 10 items per card.

WATER LEVEL F1

At any time in the program, the water level may be changed to simulate tidal fluctuations. This card increases all water depths by F1. Lower water levels can be specified by negative F1.

RAY DATA 1 NTC T AD X Y

This card establishes the starting point of a wave ray at (X,Y) with an angle AD degrees from the + X axis. NTC is any integer control number desired by the operator for ray identification. T is the wave period in seconds.

RAY DATA 2 B1 BN

This is an optional card which is used only if a ray separation factor, other than 1.00, is desired at the start of a wave ray. B1 is the Beta value at (X,Y), BN is the Beta value at an imaginary point (XN,YN) preceeding (X,Y) by a distance S along the ray.

INCREMENT S

This is an optional card which can change the initial increment used in tracing the wave ray from 4.0 to 2.0, 1.0 or 0.5 grid units. If not specified, a value of 4.0 is used. Any interval specified is a maximum and the program will automatically change to lesser values as the ray curvature increases.

TRACE RAY_

This traces wave ray specified by RAY DATA 1 and RAY DATA 2 until it goes off edge of array, runs onto beach, or strikes breakwater zone. Program will output successive X, Y coordinates of ray; wave speed, C; ray curvature, FK; ray separation factor, BETA; coefficient of refraction, FKD; The Shoaling Coefficient, D; Ray Angle, AD; and the current increment used in tracing ray, S.

Commands required if breakwaters are desired on array:

Lack of input assumes no breakwaters; one or two breakwaters may be specified.

BREAKWATER BXAS BYAS BXBS BYBS

Establishes a breakwater (A-B) from (BXAS, BYAS) to (BXBS, BYBS).

BREAKWATER BXCS BYCS BXDS BYDS

Establishes a second breakwater (C-D) from (BXCS, BYCS) to (BXDS, BYDS). The first breakwater specified is automatically designated A-B; second is C-D.

GAP

Specifies a breakwater gap from (BXBS, BYBS) to (BXCS, BYCS). If not specified, the two breakwaters are considered independent. This card can only be used if two breakwaters have previously been input to program.

SIZE GRID_ IXN IYN GRID

Sets limits of array with X from 1 to IXN and Y from 1 to IYN. If not specified, 50 by 40 is assumed. GRID is the size in feet of each grid unit of array.

Optional Breakwater Commands:

WAVE LENGTHS DIFFR WLD

This specifies the effective distance, in wave lengths, of a breakwater beyond its tip into the diffraction zone. The wave ray will be traced until it either strikes a breakwater or passes within this number of wave lengths of a breakwater tip as measured along an orthogonal to the ray. Upon striking a breakwater or breakwater zone, ray values at the point of intersection are calculated.

LENGTH DIFFRACTED WLD

Similar to WAVE LENGTHS DIFFR except that WLD, in this case, is expressed in feet. If neither diffraction distance is specified, a value of 5.0 wave lengths is assumed.

ELIMINATE BWS

This command removes all breakwaters and gaps which have been specified. New breakwaters may afterwards be specified if desired.

Input required for diffraction calculations:

The location of one breakwater (A-B), or two breakwaters (A-B) and (C-D) must be specified as called for under refraction commands.

DIFFRACT A

DIFFRACT B

DIFFRACT C

DIFFRACT D

DIFFRACT GAP

Any one of these commands specifies that diffraction calculations should take place about the designated breakwater tip. One breakwater

must have already been input if A or B is specified; two if C or D is specified.

If GAP is specified, an imaginary gap normal to the wave ray is established between points B and C. B and C must already be in the program. GAP also requires that a wave ray angle, A, be in the program. If one remains from refraction calculations, it will be used; if not, or if a change is desired, a RAY DATA 1 card must be input prior to DIFFRACT GAP.

Any changes in A must be followed by a new DIFFRACT GAP Command.

DIFFR. COORD X Y

Calculates the coefficient of diffraction and direction of wave travel at (X, Y). In addition to having already specified which breakwater tip is to be diffracted through a DIFFRACT command; a wave speed, C; wave ray angle, A; period, T; and grid size, GRID; must already have been specified. GRID must be input through a SIZE GRID Command. If present, T and A may be left unchanged from prior operations or may be specified by a RAY DATA 1 card. If present, C may be left unchanged from a prior portion of the program or must be specified by a CALC WAVE SPEED or by a WAVE SPEED Command.

Other Commands:

WAVE SPEED C

Sets C equal to a specified value.

CALC WAVE_SPEED X Y

Calculates and sets C equal to the wave speed at (X, Y).

LOCATE SHORELINE

Based on depth data, which must already have been specified, this command locates and traces the shoreline across an array. Shoreline coordinates are output.

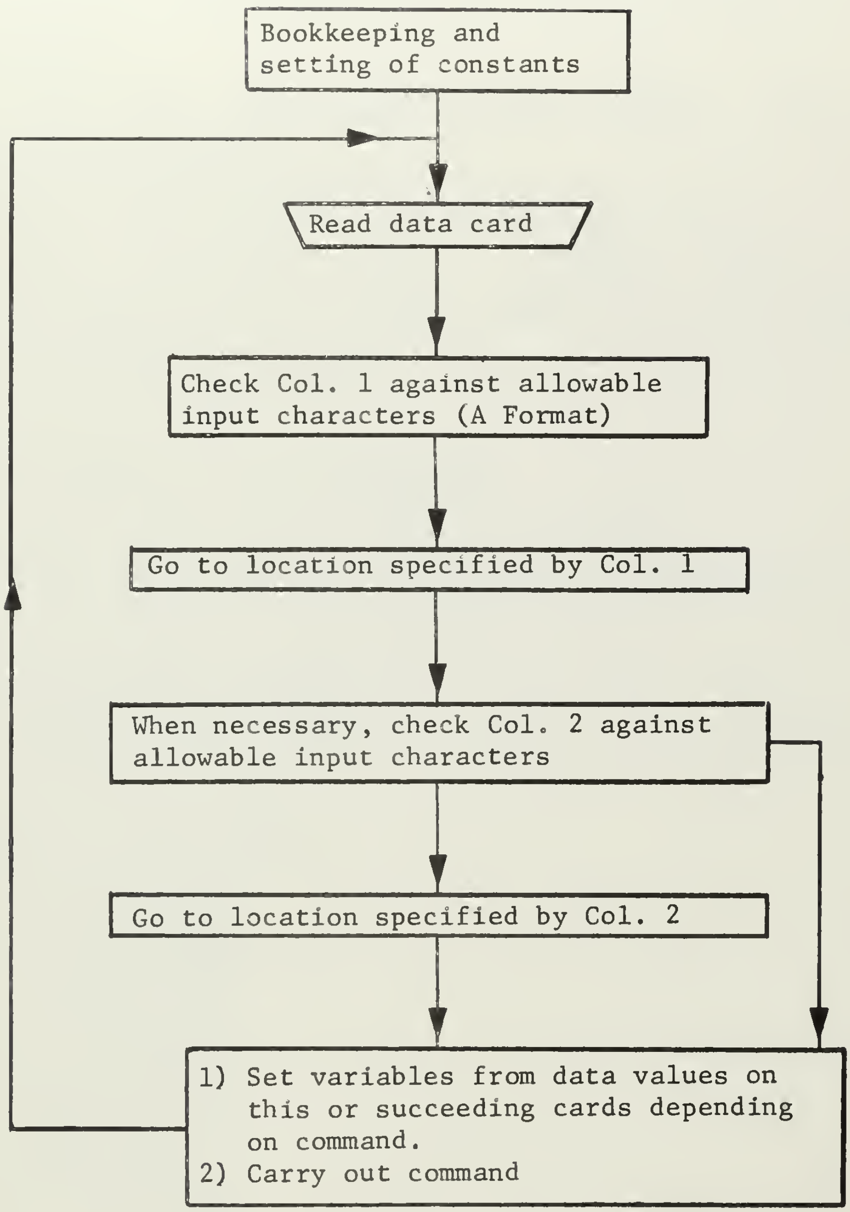
_(blank) _ N1 N2 F1 F2 F3 F4

Indicates repeat of preceeding command with new data values.

ALL DONE _

Calls for end of the program.

MACRO FLOW CHART OF MAIN



```
C     MAIN PROGRAM TO READ IN DATA AND CALL FOR APPROPRIATE ACTION.
C     PROGRAMMED BY R D SMART AT M I T CAMBRIDGE, MASS, 1967
      DIMENSION DMAT(50,40),IWD(13)
      COMMON DMAT,IXN,IYN,PI,G,GRID,X,Y,DXY,D1X,D1Y,D2X,D2Y,D2XY,
     1      A,AD,T,FK,BN,B1,B2,C,PK,PPK,CPPK,CN,DDDC,DCDX,DCDY,
     2       S,NBW,BXAS,BYAS,BXBS,BYBS,BXCS,BYCS,BXDS,BYDS,WLD
  199 FORMAT (A1,8X,A1,10X,2I5,5F10.5)
  198 FORMAT (10F6.4)
  197  FORMAT (19H TILT  INPUT ERROR ,4HWD1=,A1,6H  WD2=,A1)
  196 FORMAT (20H DEPTH ADJUSTMENT IS,F5.3, 3HFT.)
  195 FORMAT (10H ALL DONE. )
  194 FORMAT (18H  WAVE SPEED AT X= ,F5.2, 3H Y= ,F5.2, 4H IS  ,
     1      F6.2, 16H FT/S, DEPTH IS  ,F6.3,4H FT. )
  193 FORMAT (22H WAVE SPEED IS SET AT  ,F6.2,5H FT/S )
  192 FORMAT (12H  TEST CASE , I2,11H   PERIOD = ,F5.1,4HSEC.,
     1      8H  ANGLE=, F5.1, 4HDEG., 2HX=, F5.2, 2HY=, F5.2)
  191 FORMAT (20X,2I5,F10.5)
  190 FORMAT (17H BREAKWATER FROM  ,F6.2,2H,  ,F6.2,4H TO  ,F6.2,
     1          2H,  ,F6.2  )
  189 FORMAT (5H GAP  )
  188 FORMAT (24H ELIMINATED BREAKWATERS  )
  187 FORMAT (26H INITIAL RAY INCREMENT IS  ,F5.2)
  186 FORMAT (15H DIFFR DIST IS  ,F8.2, 6H FEET  )
  185 FORMAT (15H DIFFR DIST IS  ,F6.2,14H WAVE LENGTHS  )
  184 FORMAT (25H ANALYTICAL DATA FROM X=  ,I4,4H TO  ,I4,
     1          8H AND Y=  ,I4  ,4H TO  , I4  )
  183 FORMAT (9H GRID IS  ,I4,4H BY  ,I4,6H WITH  ,F8.2,
     1        15H FOOT SQUARES.  )
  182 FORMAT (19H TOPO DATA FROM X=  ,I4,4H TO  ,I4,8H AND Y=  ,
     1         I4,4H TO  ,I4)
C
C     MISCELLANEOUS BOOKKEEPING
C
      IWD(1) = -1052753856
      IWD(2) = -1035976640
      IWD(3) = -1019199424
      IWD(4) = -1002422208
      IWD(5) = -952090560
      IWD(6) =  1077952576
      IWD(7) = -650100672
      IWD(8) = -918536128
      IWD(9)= -482328512
      IWD(10)= -431996864
      IWD(11) = -750763968
      IWD(12) = -499105728
      IWD(13) = -985644992
      PI = 3.1415927
      G = 32.2
C     ASSUMED CONSTANTS.....UNTIL RESET BY OPERATOR
      DO 50 IX = 1,50
      DO 50 IY = 1,40
```

```
   50 DMAT(IX,IY) = -10000.
      NBW = 1
      B1 = 1.
      B2 = 1.
      S = 4.
      NWL = 1
      WLD = 5.
      IXN = 50
      IYN = 40
C
C   READ IN ONE COMMAND/DATA CARD
C
  100 READ (5,199) IWD1,IWD2,N1,N2,F1,F2,F3,F4
      LLS = LL
C
C   CHECK IWD1 AGAINST ALLOWABLE INPUT CHARACTERS..(A FORMAT)
C
      DO 104 LL = 1,13
      IF (IWD(LL) - IWD1) 104,106,104
  104 CONTINUE
  108 WRITE (7,197)
      GO TO 138
C
C   IF NO COMMAND IS GIVEN (BLANK), ASSUME REPEAT OF OLD COMMAND
C
  150 LL = LLS
C
C   GO TO LOCATION SPECIFIED BY COL. 1
C
  106 GO TO (138,120,137,110,126,150,128,134,114,116,140,112,142),LL
C
C   READ A DIFFRACTION COMMAND
C
  110 DO 1111 LL2 = 1,7
      IF (IWD2-IWD(LL2)) 1111,1112,1111
 1111 CONTINUE
      WRITE (7,197)
      GO TO 138
 1112 CALL RSDIFF (LL2,IWD2,F1,F2)
      GO TO 100
C
C   READ IN GRID SIZE
C
  112 IXN = N1
      IYN = N2
      GRID = F1
      WRITE (7,183) IXN,IYN,GRID
      GO TO 100
  114 IF (IWD2 - 1077952576) 1141,136,1141
C
C   READ IN GRID DATA
```

```
C
 1141 READ (5,198)((DMAT(I,J),I=IX1,IX2),J=IY1,IY2)
C    WHEN NECESSARY CONVERT FROM FATHOMS TO FEET
      IF(IWD2 + 1052753856) 100,115,100
  115 DO 1151 I=IX1,IX2
      DO 1151 J=IY1,IY2
 1151 DMAT (I,J) = 6.*DMAT(I,J)
      GO TO 100
  116 IF (IWD2 + 1002422208) 1161,1371,1161
 1161 IF(IWD2 + 482328512) 1162,1163,1162
C
C    SET DIFFRACTION IN WAVE LENGTHS
C
 1163 WLD = F1
      NWL = 1
      WRITE (7,185) F1
      GO TO 100
C
C    ADJUST WATER LEVEL
C
 1162 WRITE (7,196) F1
      DO 118 I=1,IXN
      DO 118 J=1,IYN
  118  DMAT(I,J) = DMAT(I,J) + F1
      GO TO 100
C
C    READ IN BREAKWATER COORDINATES  (2 MAX.)
C
  120 GO TO (122,124,108),NBW
C    FIRST BREAKWATER
  122 BXAS = F1
      BYAS= F2
      BXBS= F3
      BYBS=F4
      NBW = 2
      WRITE (7,190) F1,F2,F3,F4
      GO TO 100
C    SECOND BREAKWATER
  124 BXCS = F1
      BYCS = F2
      BXDS = F3
      BYDS = F4
      NBW = 3
      WRITE (7,190) F1,F2,F3,F4
      GO TO 100
C
C    SPECIFY GAP BETWEEN 2 BREAKWATERS
C
  126 IF(NBW -3) 108,127,108
  127 NBW = 4
      WRITE (7,189)
```

```
      GO TO 100
C
C   READ IN RAY DATA
C
  128 IF(IWD2 + 247447488) 130,132,130
  132  NTC = N1
      T = F1
      AD = F2
      A = AD*PI/180.
      X = F3
      Y = F4
      GO TO 100
  130 B1 = F1
      B2 = F2
      GO TO 100
C
C   CALC ANALYTICAL MATRIX
C
  138 IF (IWD2 + 750763968) 1381,1382,1381
 1382 IX1 = N1
      IX2 = N2
      DX = F1
      DM = F2
      READ (5,191) IY1,IY2,DY
      DMAT (IX1,IY1) = DM
      IXP1 = IX1 + 1
      IYP1 = IY1 + 1
      DO 1385 IX = IXP1,IX2
 1385 DMAT (IX,IY1) = DMAT(IX - 1,IY1) + DX
      DO 1387 IY = IYP1,IY2
      DMAT (IX1,IY) = DMAT(IX1,IY - 1) + DY
      DO 1387 IX = IXP1,IX2
 1387 DMAT (IX,IY) = DMAT (IX - 1,IY) + DX
      WRITE (7,184) IX1,IX2,IY1,IY2
      GO TO 100
C
C   ALL DONE.....QUIT
C
 1381 WRITE (7,195)
      CALL EXIT
C
C   CALCULATE WAVE SPEED
C
  137 NN = 1
      X = F1
      Y = F2
      CALL RSCAFK (NN)
      WRITE (7,194) X,Y,C,DXY
      GO TO 100
C
C   PRESET WAVE SPEED (FOR DIFFRACTION CALCULATIONS)
```

```
C
 1371 C = F1
      WRITE (7,193) C
      GO TO 100
C
C   TRACE RAY
C
  136 WRITE (7,192) NTC,T,AD,X,Y
      CALL RSRFTN (NWL)
C   RESET NORMAL INITIAL CONDITIONS
      B1 = 1.
      B2 = 1.
      S = 4.
      GO TO 100
C
C   CHANGE NORMAL TRACE INTERVAL FOR RAY CALCULATIONS.
C
  134 S = F1
      WRITE (7,187) F1
      GO TO 100
  140 IF (IWD2-1077952576) 1401,1402,1401
 1401 IF (IWD2 + 968867776) 1403,1404,1403
C
C   SET DIFFRACTION DISTANCE IN FEET
C
 1404 NWL = 2
      WLD = F1
      WRITE (7,186) F1
      GO TO 100
C
C   TRACE SHORELINE
C
 1403 CALL RSSHOR(DMMY)
      GO TO 100
C
C READ IN LIMITS OF TOPO DATA
C
 1402 IX1 = N1
      IX2 = N2
      READ (5,191) IY1,IY2
      WRITE (7,182) IX1,IX2,IY1,IY2
      GO TO 100
C
C   ELIMINATE BREAKWATERS
C
  142 NBW = 1
      WRITE (7,188)
      GO TO 100
      END
```

2. SUBROUTINE RSRFTN

Subroutine which traces a wave ray through shoaling water.

SUBROUTINE NAME: RSRFTN

Variables Used in Program:

COMMON Variables are listed under MAIN.

ABAR . . . . . . Ray Angle from (XN, YN) to (X, Y); an average of the angles at the two points.

AN, AMI . . . . Last 2 preceeding values of A. Used to interpolate intermediate value between AN and A.

BBW1 . . . . . Y intercept of Breakwater No. A-B.

BBW2 . . . . . Y intercept of Breakwater No. C-D.

BBW3 . . . . . Y intercept of breakwater gap.

BL . . . . Y intercept of breakwater component.

BR . . . . Y intercept of the ray.

D . . . . Shoaling Coefficient; D in text

DELS . . . . . In case of intersection, the distance from (XN, YL) as compared to the original S distance.

DELTA . . . . . Change in Ray Angle, A, from (XN, YN) to (X, Y).

DMMY . . . . . Dummy; no significance.

DN . . . . . . 2*(Deep water wave length); used to estimate deepest reasonable depth at which waves will feel bottom. At depths greater than DN, it is assumed the wave ray curvature is zero.

DN2 . . . . . . 0.5 (deep water wave length); depth at which waves are substantially affected by bottom. Between DN and DN2, only one wave ray curvature calculation is made. At depths less than DN2, up to 10 iterations are used to arrive at a final curvature.

FKBAR . . . . . Value of FK used to estimate curvature from (XN, YN) to (X, Y); usually it is an "average" value.

FKD . . . . . . Coefficient of Refraction; K_d in text.

FKL . . . . . . The last value of FK; used to check for convergence at a given (X, Y) point.

FKN, FKNMI . . Last 2 preceeding values of FK. Used to interpolate average value between FKN and FK.

FLEN . . . . . Length of diffraction zone in grid units.

FLM . . . . . . Slope of the breakwater component or zone being checked for intersection with the ray.

FLMR . . . . . Slope of the ray from (XN, YN) to (X, Y).

FLMR2 . . . . . Inverse slope of FLMR.

FMBW1 . . . . . Slope of Breakwater No. A-B.

FMBW2 . . . . . Slope of Breakwater No. C-D.

FMBW3 . . . . . Slope (x/y) of breakwater gap.

N . . . . Number of intervals moved along ray. Ray stops at N=100 as a reasonable maximum.

NDXY . . . . . Index used to determine location in program prior to breakwater intersection test; 1 if in refraction zone, 2 if in deep water.

NFKI . . . . . Counter for the number of FK iterations at a given point. If not satisfactory convergence occurs within 10 iterations, an average value is used and the ray continues.

NH. . . . . . . Intersection test index.

NN . . . . . . Index for RSCAFK to indicate the program status when called; NN=1 indicates need to have DXY calculated by RSINTP and only C calculated by RSCAFK; NN=2 indicates DXY is known, C and FK are desired.

NP . . . . . . Index for RSBETA: NP=1, at N=1; NP=2, at N greater than 1; NP=3, at a breakwater intersection.

NT . . . . . . Index to assure that, when necessary, at least 2 FK iterations are taken.

NWL . . . . . . Indication from MAIN as to whether WLD is in feet (NWL=2) or WLD is in wave lengths (NWL=1).

SOLD . . . . . Old value of S.

XDIST, YDIST . X and Y components of FLEN.

XL, YL . . . . X and Y intersection coordinates of the ray with the breakwater component.

XN, YN . . . . Preceeding (X, Y) point.

X1, X2,

Y1, Y2 . . . . X and Y limits of breakwater component.

Functions used in Program

BNEWF (AX, BX, CX, DX) Given 3 values, AX, BX, CX, function uses parabolic interpolation to calculate value of a point which is "DX" fraction of way from BX towards CX.

Summary of Program

RSRFTN is called by MAIN after a DMAT array is established and after initial ray values have been assigned, including the starting point, (X, Y), period, T, bearing angle, A, and the initial wave ray separation factors, BN and B1. RSRFTN uses iteration procedures to calculate the path of a ray through a DMAT array of varying depths. RSINTP is called to calculate water depth and bottom derivatives through the use of 3-dimensional continuous parabolic interpolation. RSCAFK is called to calculate wave speed, C, and curvature, FK, at given points; RSBETA is called to calculate the wave ray separation factor, Beta, the coefficient of refraction, K_d, and the shoaling coefficient, D. The workings of these subroutines are described in succeeding sections.

An iteration process is used to locate (X, Y) from a preceeding (XN, YN) point. Point (XN, YN), the ray curvature at that point, FKN, and the ray angle, AN, are used to estimate a location, (X, Y), of the ray some incremental distance, S, ahead. RSINTP is then called to determine bottom depth, DXY, and the bottom derivatives at the new (X, Y). These data are then used by RSCAFK to calculate C and FK at (X, Y). This new FK is then averaged with FKN to give FKBAR. This new average curvature is then used to make a revised estimate of (X, Y). At this new (X, Y), the curvature is again calculated, a new average curvature is

determined and the (X, Y) estimate is again revised. This estimating is continued until FK does not change more than 0.0005 from the FK at the preceeding (X, Y) estimate. Once (X, Y) is established, a check is made to determine if the ray from (XN, YN) to (X, Y) intersected any breakwaters or passed within any breakwater diffraction zones. If no intersection occurred, RSBETA is called to calculate Beta, K_d, and D. Upon return, the ray values at (X, Y) are printed.

The ray is then incremented to point (X, Y) which becomes (XN, YN) and the process begins again.

If at any point, DXY is negative, it is assumed a shoreline has been crossed. If the ray passes within 2 grid units of the array edges, the ray is stopped.

In water deeper than 2 wave lengths, FK is assumed to be zero and Beta remains constant while the ray is incremented along a straight line.

In depths between 0.5 and 2 wave lengths, only one calculation of FK is made and no iterations are used.

As the ray is traced shoreward, S is reduced from a maximum value of 4.0 to 2.0 as soon as FK is greater than 0.0001; again from 2.0 to 1.0 when FK is greater than 0.0005, and finally from 1.0 to 0.5 when FK is greater than 0.01. S never exceeds its old value however. Initial S values shorter than 4.0 may be specified; this will not change the reduction process.

Slope intercepts are used to determine intersection of the ray with any breakwater or breakwater diffraction zone (see MAIN). Breakwater and gap slopes and Y-intercepts are determined at the start of RSRFTN for use later as needed. Because diffraction beyond breakwater tips is considered to be orthogonal to the ray angle, these slopes and Y-intercepts must be computed individually. If intersection of a ray and

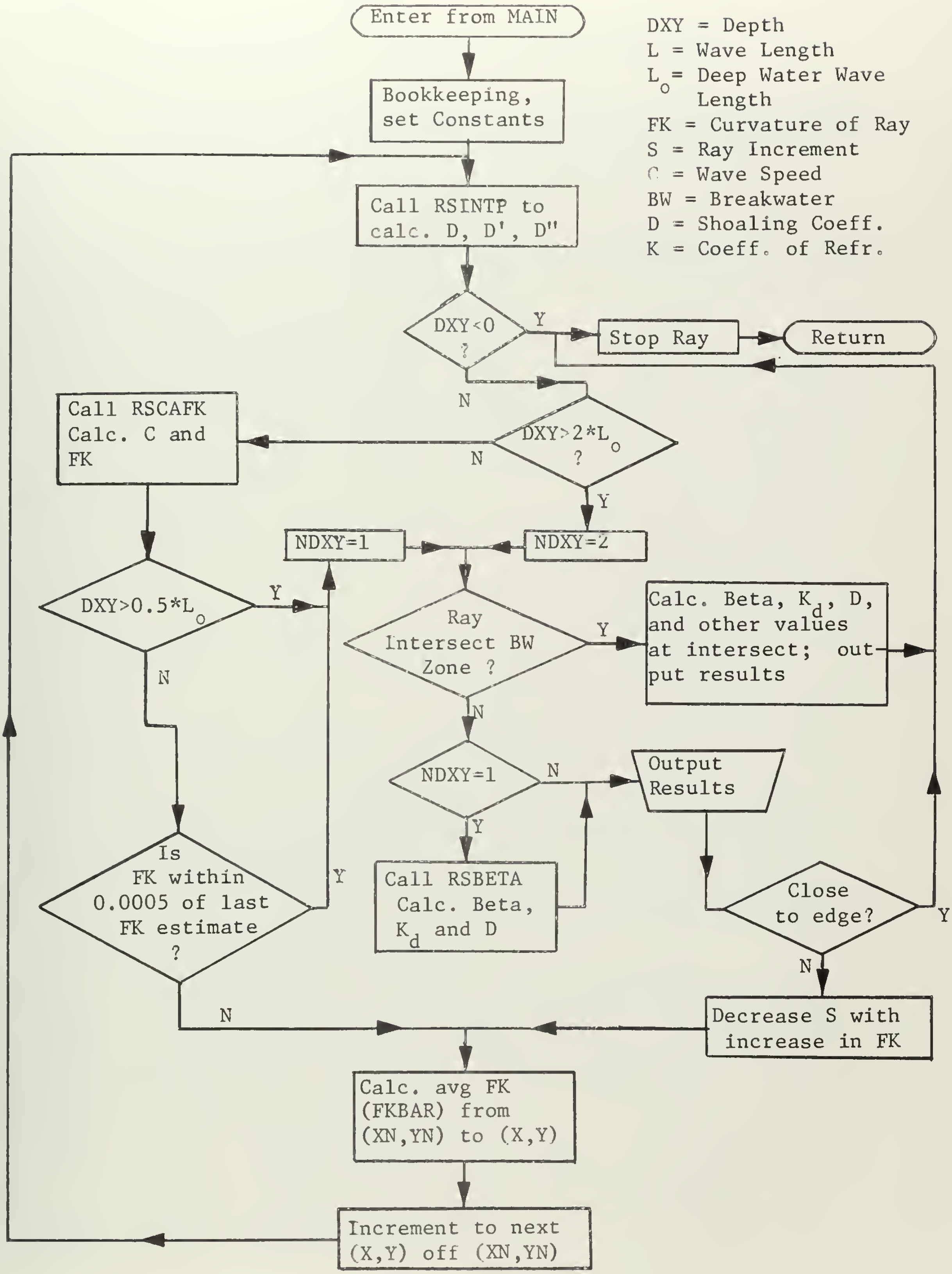
breakwater component occurs, ray values at the point of intersection are calculated directly or by using a quadratic approximation and the ray is stopped.

A few safeguards are included in the program to prevent excessive running time in case of error.

N is limited to 100 as a reasonable maximum number of iterations for a ray to require before terminating at a shoreline or an edge.

NFK1 is limited to 10 as the maximum number of iterations allowed for convergence of FK values at any one point. Upon reaching 10 iterations, a "curvature approximation" message is printed, an average FK value is used, and the ray continues.

MACRO FLOW CHAT OF SUBROUTINE RSRFTN



```
      SUBROUTINE RSRFTN (NWL)
C   SUBROUTINE WHICH WILL TRACE A WAVE RAY THROUGH SHOALING WATER.
C   BASED ON 3-DIMENSIONAL PARABOLIC INTERPOLATION.
      DIMENSION DMAT(50,40)
      COMMON DMAT,IXN,IYN,PI,G,GRID,X,Y,DXY,D1X,D1Y,D2X,D2Y,D2XY,
     1      A,AD,T,FK,BN,B1,B2,C,PK,PPK,CPPK,CN,DDDC,DCDX,DCDY,
     2       S,NBW,BXAS,BYAS,BXBS,BYBS,BXCS,BYCS,BXDS,BYDS,WLD
C
C   BOOKKEEPING AND SETTING OF CONSTANTS.
C
C   FUNCTION TO CALC INTERMEDIATE VALUE IN A SERIES
      BNEWF(AX,BX,CX,DX) = BX+(CX-AX)*DX/2.+((CX-2.*BX+AX)*DX**2)/2.
      WRITE (7,992)
  992 FORMAT (38X, 7H COEF   , 5HSHOAL )
      WRITE (7,993)
  993 FORMAT (2X,3H N  ,7H   X    ,7H   Y    ,7H   A    ,6H   C    ,
     1  6H BETA ,7H RFTN  ,5H COEF,8H  CURV  ,5H  S  ,6H DPTH )
    1 PK = T/(4.*PI)
      PPK = 2.*PI/(G*T)
      CN = G*T/(2.*PI)
      C = CN
      DN = 2.0*CN*T
      DN2 = 0.5*CN*T
      NDXY = 1
      BN = B1
      AN = A
      AM1 = AN
      XN = X
      YN = Y
      N = 1
      NP = 1
      NT = 1
C
C   SET UP CONSTANTS FOR BREAKWATER INTERSECTION EQUATIONS
C   WATCH OUT FOR INFINITE SLOPES
C   NBW = NO. OF BREAKWATERS...  1 = NONE.... 2 = 1 BW....
C                                3 = 2 BW....  4 = 2 BW + GAP
C
      GO TO (10,103,102,101),NBW
  101 IF (ABS(BXCS-BXBS) - 0.00001) 1001,1001,1002
 1001 BXCS = BXCS + 0.0001
 1002 FMBW3 = (BYCS-BYBS)/(BXCS-BXBS)
      BBW3 = BYBS - FMBW3*BXBS
  102 IF (ABS(BXDS-BXCS) - 0.00001) 1003,1003,1004
 1003 BXDS = BXDS + 0.0001
 1004 FMBW2 = (BYDS-BYCS)/(BXDS-BXCS)
      BBW2 = BYCS - FMBW2*BXCS
  103 IF (ABS(BXBS - BXAS) - 0.00001) 1005,1005,1006
 1005 BXAS = BXAS + 0.0001
 1006 FMBW1 = (BYBS-BYAS)/(BXBS-BXAS)
      BBW1 = BYAS - FMBW1*BXAS
```

```
C
C    FIND DEPTH AND BOTTOM SLOPES AT X,Y
C
   10 CALL RSINTP (DMMY)
C
C    CHECK FOR SHORELINE
       IF(DXY)100,100,13
C    CHECK TO SEE IF DEEP WATER.....DEEP = 2*WL
   13 IF (DN-DXY) 12,12,50
C
C    IF DEEP WATER..NO TRIAL AND ERROR GO ON WITH FK=0,DEL BETA = 0
C
   12 C = CN
      FKN = FK
      FK = 0.
      BN = B1
      B1 = B2
      FKD = 1.
      D = 1.
C
C    CHECK FOR BREAKWATER
C
      NDXY = 2
      GO TO 90
   19 NDXY = 1
C
C    ITERATIONS HAVE STOPPED AT THIS POINT
C    WRITE OUT RESULTS
C
   20 AD = A * 180.0/ PI
      WRITE  (7,998) N,X,Y,AD,C,B1,FKD,D,FK,S,DXY
  998 FORMAT (1X,I3,3F7.3,F6.2,F6.3,F7.3,F5.2,F8.4,F5.2,F6.2)
C
C    CHECK FOR EDGES
C
   22 IF ((X-2.)*(FLOAT(IXN) - 1. - X)) 100,100,23
   23 IF  ((Y-2.)*(FLOAT(IYN) - 1. - Y)) 100,100,24
C    SAFETY STOP FOR RAY AT N = 100
   24 IF (N-100) 30,30,100
C
C    S CHECK  CHANGE S VALUE DEPENDING ON CURVATURE OR RAY.
C    IF FK L.T. 0.0001 USE S = 4.0 GRID UNITS...THEN USE S = 2.0 UNTIL
C    FK G. T . 0.005, THEN USE S = 1.0 UNTIL FK G. T. 0.01,
C    THEN USE S = 0.5
C    DO NOT EXCEED OLD S VALUE
C    UPON CHANGING S ADJUST B2 TO INTERMEDIATE VALUE.
C
   30 SOLD = S
      IF (S-3.999) 202,202,210
  202 IF (S-1.999) 204,204,220
  204 IF (S-0.999) 300,300,230
```

```
  210  IF (ABS(FK) -0.0001) 300, 300, 220
  220  IF (ABS(FK) - 0.005) 222,222,230
  222 IF (SOLD - 2.001) 300,300,224
  224 S = 2.
  225 B2 = BNEWF(BN,B1,B2,0.5)
      GO TO 300
  230  IF (ABS(FK) - 0.01) 232,232,240
  232 IF ((SOLD - 1.001) - 0.0001) 300,300,234
  234 S = 1.0
      IF ((SOLD -2.001) - 0.0001) 225,225,236
  236 B2 = BNEWF(BN,B1,B2,0.25)
      GO TO 300
  240 S = 0.5
      IF ((SOLD -1.001) - 0.0001) 225,225,244
  244 IF ((SOLD - 2.001) - 0.0001) 236,236,248
  248 B2 = BNEWF(BN,B1,B2,0.125)
C
C    INCREMENT TO NEXT POINT BASED ON BEST FKBAR ESTIMATE.
C    WHEN INITIALLY INCREMENTING, SAVE LAST POINT AND ASSOCIATED
C             VALUES AS A JUMPING OFF POINT FOR NEXT POINT.
C
  300 AM1 = AN
      AN = A
      XN = X
      YN = Y
      FKNM1 = FKN
      FKN = FK
      NT = 1
      N = N+1
      FKBAR = FK
C
C    GO ON TO NEXT X,Y POINT BASED ON BEST FKBAR ESTIMATE.
C
   40 DELTA = S * FKBAR
      A = AN + DELTA
      ABAR = AN + DELTA/2.
      X = XN + S*COS(ABAR)
      Y = YN + S*SIN(ABAR)
      GO TO 10
C
C    CALCULATE WAVE SPEED AND CURVATURE AT POINT (X,Y)
C
   50 IF (NT-1) 52,52,80
C    MAKE FIRST ESTIMATE OF FK
   52 NFKI = 1
      NT = 2
C    CALC WAVE SPEED AND CURVATURE (C AND FK)
   54 NN = 2
      CALL RSCAFK(NN)
C    N = 1 POINT ONLY REQUIRES AN FK VALUE.
      IF (N-1) 920,920,541
```

```
  541 GO TO (70,70,82),NT
   70 FKL = FK
      FKBAR = BNEWF(FKNM1,FKN,FK,0.5)
C
C    MAKE JUST ONE PASS IN ))INTERMEDIATE)) DEPTHS
C       INTERMEDIATE = L.T. 2WL, OR G.T. 0.5WL)
C
      IF (DXY - DN2) 40,90,90
C
C    IN SHALLOW WATER REVISE FK ESTIMATES UNTIL TWO AGREE WITHIN
C            0.0005...MINIMUM OF TWO ESTIMATES ARE NEEDED.
C
   80 NT = 3
      NFKI = NFKI +1
      GO TO 54
C    CHECK FOR CONVERGENCE....DONT TRY MORE THAN 10 TIMES
   82 IF ((ABS(FKL-FK))-0.0005) 90,90,84
   84 IF (NFKI - 10) 70,70,86
   86 WRITE (7,995)
  995 FORMAT (24H CURVATURE APPROXIMATED   )
      FK = (FKL + FK)/2.
C
C    CHECK FOR INTERSECTION WITH BREAKWATER OR BREAKWATER ZONE OF
C            OF INFLUENCE....IF THERE ARE ANY BREAKWATERS...
C
   90 IF (NBW - 2) 92,255,255
  255 IF (ABS(X-XN) -0.00001) 2551,2551,256
 2551 IF (ABS(Y-YN)-0.00001)92.92,256
C    CHECK FOR BREAKWATER  NO.A-B
  256 FLMR = (Y-YN)/(X-XN)
      BR = YN - FLMR*XN
      FLM = FMBW1
      BL = BBW1
      X1 = BXAS
      X2 = BXBS
      NH = 1
      GO TO 292
C    CHECK FOR DIFFRACTION ZONE OFF A END.
  257 IF (GRID-0.0001) 2574,2574,2575
 2574 WRITE (7,299)
  299 FORMAT(52H PLEASE TELL ME MY GRID SIZE,OR I WONT WORK FOR YOU.)
      GO TO 100
 2575 GO TO (2571,2572),NWL
 2571 FLEN = WLD * C * T / GRID
      GO TO 2573
 2572 FLEN = WLD / GRID
 2573 IF (ABS(FLMR)-0.0001) 2578,2578,2576
 2578 XDIST = 0.
      GO TO 2577
 2576 FLMR2 = -1./FLMR
      XDIST = FLEN/(1.+FLMR2**2)**0.5
```

```
 2577 X2 = BXAS - XDIST
      YDIST = FLEN/(1.+FLMR**2)**0.5
      IF (BYBS-BYAS) 262,262,260
  260 YDIST = -YDIST
  262 Y2 = BYAS + YDIST
      X1 = BXAS
      Y1 = BYAS
      NH = 2
      GO TO 290
C   CHECK TO SEE HOW MANY BREAKWATERS THERE ARE.
  264 GO TO (92,266,266,272),NBW
C   CHECK FOR DIFFRACTION ZONE OFF END B
  266 X2 = BXBS + XDIST
      Y2 = BYBS - YDIST
      X1 = BXBS
      Y1 = BYBS
      NH= 3
      GO TO 290
C   CHECK FOR DIFFRACTION ZONE OFF END C.
  268 IF (NBW-2)92,92,270
  270 X2 = BXCS - XDIST
      IF ((BYDS-BYCS)*(BYBS-BYAS)) 2701,2702,2702
 2701 YDIST = -YDIST
 2702 Y2 = BYCS + YDIST
      X1 = BXCS
      Y1 = BYCS
      NH = 4
      GO TO 290
C   CHECK FOR INTERSECTION WITH GAP.
  272 FLM = FMBW3
      BL = BBW3
      X1 = BXBS
      X2 = BXCS
      NH = 5
      GO TO 292
C   CHECK FOR INTERSECTION WITH BREAKWATER NO.C-D
  274 FLM = FMBW2
      BL = BBW2
      X1 = BXCS
      X2 = BXDS
      NH = 6
      GO TO 292
C   CHECK FOR DIFFRACTION ZONE OF END D.
  276 X2 = BXDS + XDIST
      Y2 = BYDS - YDIST
      X1 = BXDS
      Y1 = BYDS
      NH = 7
C   USE SLOPE INSERSECTION (M*X + B) TO SEE IF RAY AND
C             BREAKWATER COMPONENT INTERSECT.
  290 IF (ABS(X2-X1) -0.00001) 2901,2902,2902
```

```
2901 X2 = X2 + 0.0001
2902 FLM = (Y2-Y1)/(X2-X1)
     BL = Y1 - FLM*X1
 292 IF (ABS(FLMR-FLM) - 0.00001) 2921,2922,2922
2921 FLMR = FLMR + 0.0001
2922 XI = (BL-BR)/(FLMR-FLM)
     IF((X-XI)*(XI-XN))294,293,293
 293 IF((X2-XI)*(XI-X1)) 294,296,296
 294 GO TO (257,264,268,274,274,276,92),NH
C
C   IF THEY DO INTERSECT CALCULATE POINT OF INTERSECTION VALUES,
C          INCLUDING RAY ANGLE, RAY SEPARATION FACTOR,
C             COEFFICIENT OF REFRACTION AND SHOALING FACTOR.
C
 296 YI = (XI-XN)*(Y-YN)/(X-XN) + YN
     DELS = ((XI-XN)**2 + (YI-YN)**2)**0.5/S
     B1 = BNEWF(BN,B1,B2,DELS)
     A = BNEWF(AM1,AN,A,DELS)
     AD = A * 180./PI
     FKD = (1./B1)**0.5
     X = XI
     Y = YI
     NN = 1
     CALL RSCAFK(NN)
     IF (NP-1) 2962,2962,2961
2962 D = 1.
     GO TO 2963
2961 NP = 3
     CALL RSBETA (NP,D,FKD)
2963 WRITE (7,298) XI,YI
     WRITE (7,297)
     WRITE (7,993)
     WRITE (7,998) N,X,Y,AD,C,B1,FKD,D,FK,S,DXY
 298 FORMAT (41H INTERSECTION WITH BREAKWATER ZONE AT X=   ,
    1        F6.3,4H Y=   ,F6.3)
 297 FORMAT (36H RAY VALUES AT INTERSECTION FOLLOW.   )
     GO TO 100
  92 GO TO (921,19),NDXY
C
C   CALCULATE BETA
C
 920 FKN = FK
 921 CALL RSBETA (NP,D,FKD)
     GO TO 20
C
C END OF RAY CALCULATIONS.
 100 WRITE (7,994)
 994 FORMAT (13H  RAY STOPPED )
     RETURN
     END
```

3. SUBROUTINE RSINTP

Subroutine which calculates depth and partial derivatives of the bottom.

SUBROUTINE NAME: RSINTP

Variables used in Program

COMMON variables are listed under MAIN

DMMY. . . . . . Dummy variables, no significance

JX, JY . . . . Integer X and Y; not rounded.

DLX, DLY . . . Decimal fraction of (X-JX) and (Y-JY).

IX, IY. . . . . (JX-2) and (JY-2).

NX, NY. . . . . Counters for X and Y.

IXNY, IYNY. . . X and Y grid points.

D(NY) . . . . . Bottom depth at location DMAT(IXNY, IYNY).

DD (NX) . . . . Bottom depth at (IXNX, Y).

DD1 (NX). . . . $\partial d/\partial Y$ at (IXNX, Y).

DD2 (NX). . . . $\partial^2 d/\partial y^2$ at (IXNX, Y).

Functions used in Program

DEPTHF(A1, A2, A3, A4, DELTA)

Calculates data value at point of interpolation. Given four equally spaced data values, A1, A2, A3, and A4, this function uses continuous parabolic interpolation, as described in the text, to calculate a data value at point DELTA. DELTA is that fraction of the distance from point A2 towards A3 at which we desire data values.

DEP1F (A1, A2, A3, A4, DELTA)

Calculates slope of curve at point of interpolation using continuous parabolic interpolation.

DEP2F (A1, A2, A3, A4, DELTA)

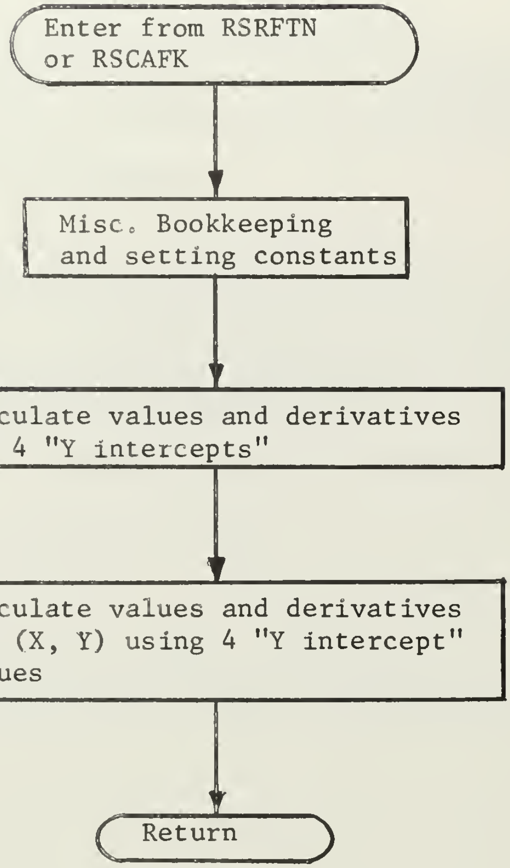
Calculates second derivative of curve at point of interpolation using continuous parabolic interpolation.

Summary of Program

Continuous parabolic interpolation is reduced to three functions; one to calculate a data value; one to calculate a slope; and one to calculate a second derivative. All values are at any abcissa value between the center two of four equally space data points.

With (X,Y) located within the center grid of a 4 x 4 data array, these functions are used by 4 interpolating parabolas, parallel to the Y axis, to define 4 points on a plane parallel to the X axis and passing through (X, Y). Using these 4 new points, and their derivatives when necessary, the same functions are then used by an interpolating parabola parallel to the X axis, and passing through (X, Y) to completely define (X, Y) and all desired derivatives of a surface through (X, Y). For a detailed explanation of this process see Section II-E (C) of text.

MACRO FLOW CHART OF SUBROUTINE RSINTP



```
      SUBROUTINE RSINTP (DMMY)
C
C   SUBROUTINE CALLED BY RSRFTN OR RSCAFK WHICH WILL USE
C          3-DIMENSIONAL PARABOLIC INTERPOLATION TO CALCULATE
C            DEPTH,BOTTOM SLOPES,AND OTHER PARTIAL DERIVATIVES
C              OF BOTTOM.
C
      DIMENSION DMAT(50,40),D(4),DD(4),DD1(4),DD2(4)
      COMMON DMAT,IXN,IYN,PI,G,GRID,X,Y,DXY,D1X,D1Y,D2X,D2Y,D2XY,
     1      A,AD,T,FK,BN,B1,B2,C,PK,PPK,CPPK,CN,DDDC,DCDX,DCDY,
     2       S,NBW,BXAS,BYAS,BXBS,BYBS,BXCS,BYCS,BXDS,BYDS,WLD
C
C   BOOKKEEPING AND SETTING OF CONSTANTS.
C
C   FUNCTIONS FOLLOW
      DEPTHF(A1,A2,A3,A4,DELTA) = A2+(DELTA/2.)*(A3-A1)-(DELTA**2/2.)
     1      *(A4-4.*A3+5.*A2-2.*A1)+(DELTA**3/2.)*(A4-3.*A3+3.*A2-A1)
      DEP1F(A1,A2,A3,A4,DELTA)= (A3-A1)/2.-(DELTA/2.)*(A4-5.*A3+7.*A2
     1    -3.*A1) + (DELTA**2) * (A4-3.*A3+3.*A2-A1)
      DEP2F(A1,A2,A3,A4,DELTA)=A3-2.*A2+A1+DELTA*(A4-3.*A3+3.*A2-A1)
      JY=Y
      DLY = Y-(FLOAT(JY))
      JX = X
      DLX = X - (FLOAT(JX))
      IX = JX -2
      IY = JY - 2
C
C   INTERPOLATE 4 TIMES PARALLEL TO Y AXIS FIRST......
C     CALCULATE DEPTHS AND DERIVATIVES FOR Y-INTERCEPTS OF (X,Y)
C
      DO 200 NX=1,4
      IXNX = IX+NX
      DO 180 NY=1,4
      IYNY = IY + NY
  180 D(NY) = DMAT(IXNX,IYNY)
      DD(NX) = DEPTHF(D(1), D(2), D(3), D(4), DLY)
      DD1(NX)= DEP1F (D(1), D(2), D(3), D(4), DLY)
  200 DD2(NX)= DEP2F (D(1), D(2), D(3), D(4), DLY)
C
C   USE RESULTS OF LAST 4 INTERPS TO CALC VALUES AT (X,Y)
C
      DXY = DEPTHF(DD(1),DD(2),DD(3),DD(4),DLX)
      D1Y = DEPTHF (DD1(1),DD1(2),DD1(3),DD1(4),DLX)
      D2Y = DEPTHF(DD2(1),DD2(2),DD2(3),DD2(4),DLX)
      D1X = DEP1F (DD(1),DD(2),DD(3),DD(4),DLX)
      D2X = DEP2F (DD(1),DD(2),DD(3),DD(4),DLX)
      D2XY=DEP1F   (DD1(1),DD1(2),DD1(3),DD1(4),DLX)
  400 RETURN
      END
```

4. SUBROUTINE RSBETA

Subroutine which calculates the ray separation factor (BETA), the coefficient of refraction, K_d, and the shoaling coefficient, D, for a ray.

SUBROUTINE NAME: RSBETA

Variables used in Program:

COMMON Variables are lised under MAIN.

CG . . . . . . Wave group velocity.

CGN . . . . . . Deep water wave group velocity.

D . . . . Shoaling coefficient; D in text.

D2CDX2 . . . . $\partial^2 C/\partial X^2$ in text.

D2CDY2 . . . . $\partial^2 C/\partial Y^2$ in text.

D2CDXY . . . . $\partial^2 C/\partial X \partial Y$ in text.

D2DDC2 . . . . $d^2 d/dC^2$ in text.

FKD . . . . . . Coefficient of Refraction; K_d in text.

FPD . . . . . . Convenient summation.

PP, QQ . . . . p and q of equation for B2 shown under summary of program.

Summary of Program

After the ray trace subroutine, RSRFTN, has determined an (X, Y) coordinate, RSBETA is called to use a "Finite Difference" method, (See

text, Section II-E (d)), for projecting from the two most recent Beta values to the next succeeding value.

The program requires 2 initial Beta values to start, one at the present (X, Y) point, B1, and one at the preceeding (XN, YN) point, BN. All points must be equally spaced.

Each time called, RSBETA increments itself from the prior BN, B1 values and calculates the succeeding B2 using

$$B2 = \left(\frac{4-2\,q\,S^2}{2+p\,S}\right) B1 + \left(\frac{p\,S-2}{2+p\,S}\right) BN$$

It must be kept in mind that this method is projecting an estimated Beta value one step ahead of ray tracing. One return from RSBETA, B1 is the present (X, Y) Beta value.

In addition, the program calculates the Shoaling Coefficient at a point by using

$$D = \left(\frac{CGN}{CG}\right)^{1/2}$$

where

$$CG = 0.5 * C * \left(1 + \frac{4\,\pi\,d/L}{\sinh\,(4\,\pi\,d/L)}\right)$$

and CGN is the deep water value of CG.

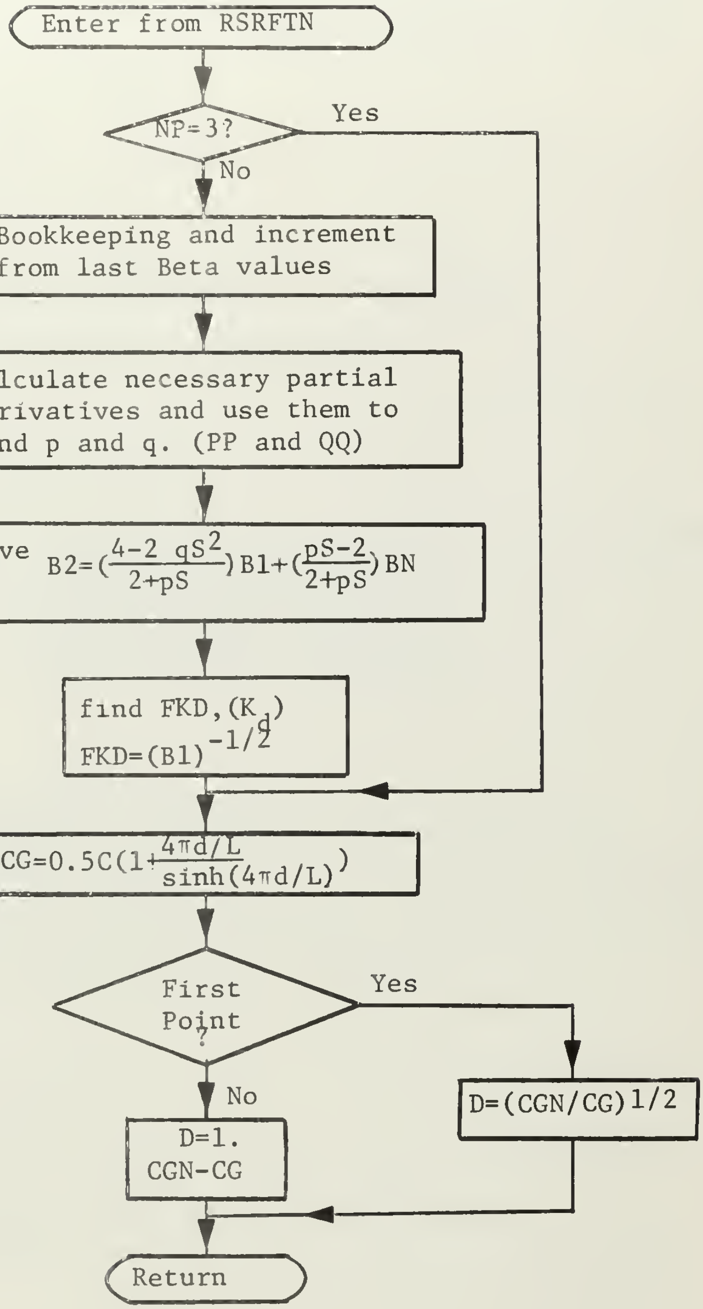
It also uses the Beta value to calculate the coefficient of refraction at a point.

$$K_d = (\mathrm{Beta})^{-1/2}$$

In program notation this is:

$$FKD = (B1)^{-1/2}$$

MACRO-FLOW CHART OF SUBROUTINE RSBETA



```
      SUBROUTINE RSBETA (NP,D,FKD)
C   SUBROUTINE TO CALCULATE RAY SEPARATION FACTOR (BETA) FOR
      DIMENSION DMAT(50,40)
      COMMON DMAT,IXN,IYN,PI,G,GRID,X,Y,DXY,D1X,D1Y,D2X,D2Y,D2XY,
     1      A,AD,T,FK,BN,B1,B2,C,PK,PPK,CPPK,CN,DDDC,DCDX,DCDY,
     2       S,NBW,BXAS,BYAS,BXBS,BYBS,BXCS,BYCS,BXDS,BYDS,WLD
C
C   INCREMENT BETA TO PRESENT POINT
C
      GO TO (700,700,705),NP
  700 BN = B1
      B1 = B2
C
C   STILL IN RELATIVELY DEEP WATER...BETA = 1.000000
C
      IF (ABS(CPPK-1.0)-0.0000001) 704,702,702
  704 B2 = 1.
      GO TO 710
C
C   CALCULATE NECESSARY DERIVATIVES
C
  702 D2DDC2 = PK*(2.*(PPK/(1.+CPPK) +PPK/(1.-CPPK)) + C*(PPK**2/
     1        (1.-CPPK)**2 - PPK**2/(1.+CPPK)**2))
      D2CDX2 = D2X/DDDC - (D2DDC2*D1X**2)/DDDC**3
      D2CDY2 = D2Y/DDDC - (D2DDC2*D1Y**2)/DDDC**3
      D2CDXY = D2XY/DDDC - (D2DDC2*D1X*D1Y)/DDDC**3
C
C   SOLVE BETA EQUATION
C
      PP = -(DCDX*COS(A) + DCDY*SIN(A))/C
      QQ =  (D2CDX2*(SIN(A)**2)- (D2CDXY*2.*SIN(A)*COS(A))
     1       + (D2CDY2*(COS(A))**2))/C
      B2 = ((PP*S-2.)/(PP*S+2.))*BN + ((4.-2.*QQ*S**2)/(PP*S+2.))*B1
C   ON RETURN PRINT OUT B1 AS BETA FOR POINTS X,Y
C
C   CALULATE COEFFICIENT OF REFRACTION.   (FKD)
C
      FKD = (1./B1)**0.5
C
C   CALCULATE SHOALING COEFFICIENT. (D)
C
  705 FPD = 4.*PI*DXY/(C*T)
      CG = 0.5*C*(1+(FPD/(0.5*(EXP(FPD)-EXP(-FPD)))))
      GO TO (706,708,708),NP
  706 CGN = CG
      D = 1.
      NP = 2
      GO TO 710
  708 D = (CGN/CG)**0.5
  710 RETURN
      END
```

5. SUBROUTINE RSCAFK

Subroutine which calculates wave velocity and ray curvature at any given (X, Y) point.

SUBROUTINE NAME: RSCAFK

Variables Used in Program

COMMON variables are listed under MAIN.

CL . . . . . . Wave speed used to check convergence of iterations.

LL . . . . . . Index; no significance.

NN . . . . . . Index to indicate program status. If NN=1, call RSINTP to determine DXY, then only calculate C. If NN=2, use present DXY and calculate both C and FK.

Summary of Program

When called with NN=1, RSCAFK calls RSINTP to determine water depth, then determines wave speed using an iteration process to solve:

$$C = \frac{gL}{2\pi} \tanh \frac{2\pi d}{L}$$

The program then returns.

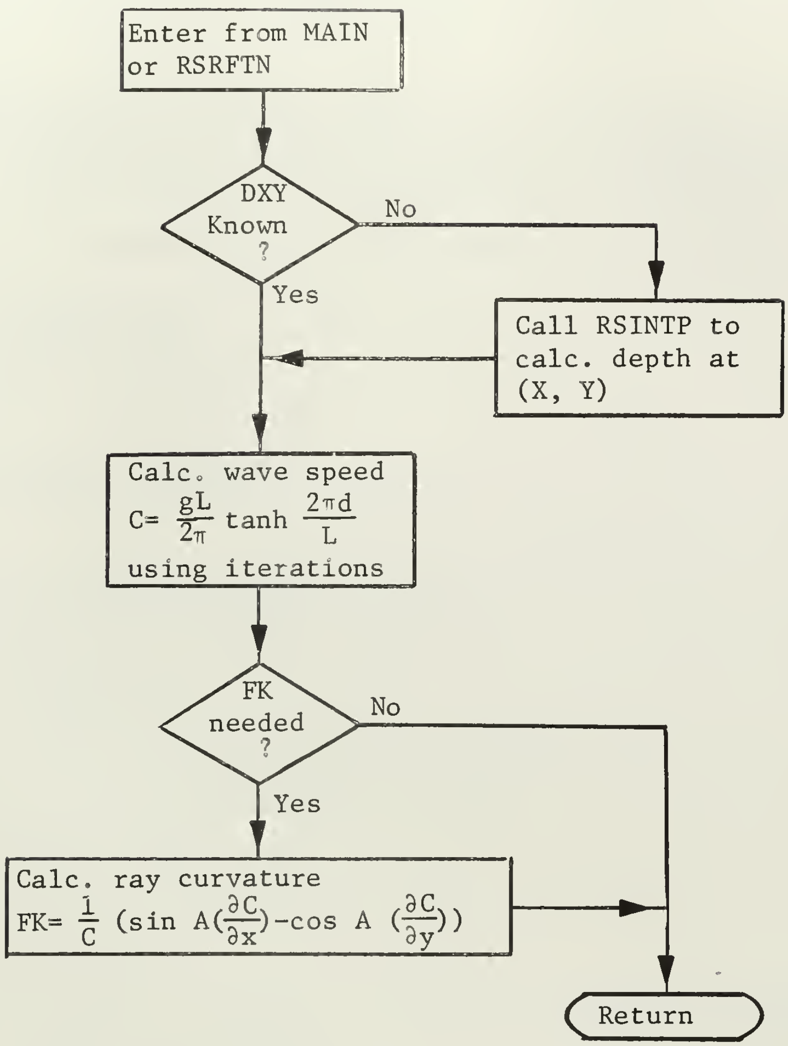
When called with NN=2, depth data have already been determined. Using these data, it determines wave speed then goes on to determine ray curvature using

$$FK = \frac{1}{C} \left(\sin A \left(\frac{\partial C}{\partial X}\right) - \cos A \left(\frac{\partial C}{\partial Y}\right)\right)$$

The program then returns.

As a safeguard, a maximum of 80 iterations are allowed for C to reach successive values within 0.0005 of each other; otherwise a "Wave Speed Approximation" message is printed and the program continues using an average C value.

MACRO FLOW CHAT OF SUBROUTINE: RSCAFK



```
      SUBROUTINE RSCAFK (NN)
C
C    SUBROUTINE WHICH CALCULATES WAVE VELOCITY AND RAY CURVATURE
C            AT ANY GIVEN (X,Y) POINT
C    USUALLY CALLED BY RSRFTN DURING RAY TRACE COMPUTATIONS,  BUT
C          MAY BE CALLED BY MAIN JUST TO CALCULATE WAVE VELOCITY
C           AT ANY SPECIFIED LOCATION.
C
      DIMENSION DMAT (50,40)
      COMMON DMAT,IXN,IYN,PI,G,GRID,X,Y,DXY,D1X,D1Y,D2X,D2Y,D2XY,
     1      A,AD,T,FK,BN,B1,B2,C,PK,PPK,CPPK,CN,DDDC,DCDX,DCDY,
     2       S,NBW,BXAS,BYAS,BXBS,BYBS,BXCS,BYCS,BXDS,BYDS,WLD
C
C    IF CALLED BY MAIN, NEED TO HAVE DEPTH CALCULATED FIRST.
C
      GO TO (53,55),NN
   53 CALL RSINTP (DMMY)
      CN = G * T / (2.*PI)
      C = CN
C
C    CALCULATE WAVE SPEED,C.
C
   55 CL = C
      DO 56 LL = 1,80
      C = CN * TANH((2.*PI*DXY)/(T*CL))
      IF (ABS(CL-C) - 0.0005) 58,58,56
   56 CL = (C + CL)/2.
      WRITE (7,999)
  999 FORMAT (26H WAVE SPEED APPROXIMATED.   )
C
C    IF CALLED BY MAIN...GO BACK TO MAIN
C
   58 GO TO (59,57),NN
C
C    CALCULATE CURVATURE OF RAY (= FK )
C
   57 CPPK = C * PPK
      DDDC = (PK)*((2.*CPPK)/(1.-CPPK**2)+ALOG  (1.+CPPK)
     1      - ALOG  (1.-CPPK))
      DCDX = D1X/DDDC
      DCDY = D1Y/DDDC
      FK = (1./C)*((SIN(A))*DCDX - (COS(A))*DCDY)
   59 RETURN
      END
```

6. SUBROUTINE RSDIFF

Subroutine to calculate Coefficient of Diffraction and Direction of Wave Front Travel at end of semi-infinite breakwaters or at a breakwater gap.

SUBROUTINE NAME: RSDIFF

<u>Variables used by Program</u>:

COMMON variables are listed under MAIN.

AA, BB . . . . A, B in text.

AA1, BB1 . . . Used to save "half" solution across breakwater gap.

ABW . . . . . . Bearing of breakwater or gap, in radians, relative to X axis.

ABW1 . . . . . ABW value used for direction checks.

AP . . . . . . Bearing from breakwater tip or center of gap to (X, Y) and relative to X axis.

A1, A2
B1X, B2X . . . Interim summations.

BXA, thru
BYD . . . . . . For diffraction at one end of breakwater, the desired end is assigned coordinates, (BXB, BYB); the other end is assigned coordinates (BXA, BYA). For diffraction at a gap, imaginary coordinates BXA through BYD are assigned based on an imaginary gap normal to the wave ray (see text).

CC . . . . C in text, part of Fresnel Integral.

CDIF . . . . . Coefficient of diffraction.

CTM, CTP . . . cos (TH-THN), cos (TH+THN) respectively.

DADTH, DADR . . $\partial A/\partial \theta$ and $\partial A/\partial r$ in text.

DADX, DADZ . . $\partial A/\partial X$, $\partial A/\partial Z$ in text.

DADZ1, DADXL,
DBDZ1, DBDX1 . Used to save "half" solution across breakwater gap.

DA1, DA2,
DB1, DB2 . . . Interim summations.

DBDTH, DBDD . . $\partial B/\partial \theta$ and $\partial B/\partial r$ in text.

DBDX, DBDZ . . $\partial B/\partial X$, $\partial B/\partial Z$ in text.

DFM, DFP,
DU1, etc. . . . Appropriate derivatives used to make passes through similar equations without rewriting equations.

DFMDR, DFPDR . $\partial FM/\partial r$, $\partial FP/\partial r$ in text.

DFMDTH,
DEPDTH . . . . $\partial FM/\partial \theta$, $\partial FP/\partial \theta$ in text.

DG . . . . Gap length.

DIR . . . . Angle of wave travel in radians.

DIRD . . . . . Angle of wave travel in degrees.

DSIG2R . . . . $\partial \sigma'/\partial r$; also $\partial \sigma/\partial r$; see Note.
Note: Same calculations are needed with both σ and σ'. A first pass is made through calculations (M=1) using σ values although the variable name indicates σ' values.

Upon completion of calculations σ, results are saved, σ' values are inserted, and a second pass is made (M=2). First pass, with σ, calculates U1, W1 and their derivatives; second pass, with σ', calculates U2, W2 and their derivatives.

DSIG2T . . . . $\partial\sigma'/\partial\theta$; also $\partial\sigma/\partial\theta$; see Note following DSIG2R.

DT . . . . . . Half width of imaginary gap normal to wave ray.

DU2DR,

DW2DR . . . . . $\partial U2/\partial r$, $\partial W2/\partial r$ or $\partial U1/\partial r$, $\partial W1/\partial r$. See note under DSIG2R.

DU2DTH,

DW2DTH . . . . $\partial U2/\partial\theta$, $\partial W2/\partial\theta$ or $\partial U1/\partial\theta$, $W1/\partial\theta$.

DX, DY . . . . Various X and Y differences used locally in various parts of program; no significance.

DXT, DYT . . . Local subtotal; no significance.

FACFP,

FBCFB . . . . . cos (FP), sin (FP) in text.

FACFPX,

FBCFPX . . . . $\partial(\cos(FP))/\partial X$, $\partial(\sin(FP))/\partial X$ in text.

FACFPZ,

FBCFPZ . . . . $\partial(\cos(FP))/\partial Z$, $\partial(\sin(FP))/\partial Z$ in text.

FF1, FF2 . . . Interim summations.

FLTK . . . . . K in text; 2 *PI/(C*T).

FM, FP . . . . FM, FP in text; kr cos (TH-THN), kr cos (TN+THM) respectively.

FN . . . . n in text, index for Fresnel summations.

FT . . . . t in text; $\pi\sigma^2/_2$

F1, F2 . . . . Variables from MAIN; coordinate locations (X, Y).

IWD2 . . . . . Alphameric variable from MAIN; the end of breakwater which is being diffracted, A, B, C, or D.

LL . . . . Index passed from MAIN which indicates desired action by subroutine. If LL=1, 2, 3, or 4, diffraction is at A, B, C, or D end of breakwater respectively. If LL=5, diffraction is at Gap. If LL=6 or 7, perform diffraction for given coordinates.

M . . . . Index; see Note following DSIG2R.

N . . . . Index (1, 2, or 3) used to return to correct location in program after calculating a bearing.

NGAP . . . . . NGAP=1 for diffraction at the end of a breakwater. NGAP=2 for gap diffraction with higher values being assigned during the process of gap solution in order to keep track of the status of calculations.

NGAPS . . . . . Original value of NGAP to be restored at end of program.

NN . . . . NN=1 if imaginary gap is being established at start of program; NN=2 at end of program when gap is being restored for future use.

NQ . . . . Index to indicate whether angle AP is greater than angle A (NQ=2), or less (NQ=1).

R . . . . r in text; distance from (X, Y) to either a breakwater tip or center of gap.

SI	σ or σ'.
SI4	σ^4.
SIGI	σ in text.
SIG2	σ' in text.
SS	S in text. Part of Fresnel Integral.
TERM	Terms of summation series.
TH	Angle from breakwater to (X, Y) as measured at breakwater tip.
THN	Angle from breakwater to ray of incident wave as measured at tip.
THD, THND	TH and THN in degrees.
THX	0.5 $(\theta+\theta_0)$ or 0.5 $(\theta-\theta_0)$; no significance.
TH11	Used to save "half" solution across breakwater gap.
U1, U2, W1, W2	Same as text.
XC, YC	Coordinates of center of gap.
ZZ	0.5 (2*R/(C*T)); no significance.
Z2	Convenient summation. Used in sin and cos approximations of Fresnel Integrals.

Summary of Program

Prior to starting diffraction calculations, incident wave data must be input to the program as previously described in the section on MAIN commands.

In order to solve a diffraction problem, RSDIFF is first called by MAIN after a DIFFRACT Command has been read. Depending on whether told to diffract breakwater end A, B, C, D, or Gap, index LL is set to 1-5 respectively by MAIN. Then RSDIFF is called to assign breakwater coordinates (BXB, BYB) to the end of a breakwater at which diffraction is to take place and assigns BXA, BYA to the other end. BXAS through BYDS are never disturbed and are kept for later use if desired. The main portion of RSDIFF is such that it always computes diffraction about the B end of an A-B breakwater. When called to diffract a gap, the program first determines an imaginary gap, (see Text Section III, B) which is normal to the incident wave ray and sets index NGAP=2 to indicate a gap problem. Control then returns to the main program.

When later called by MAIN after a DIFFR COORD command has been read, RSDIFF calculates the coefficient of diffraction and direction of wave travel at (X, Y) and about the previously designated tip or gap.

The main portion of the RSDIFF program is arranged to calculate values needed to solve a standard semi-infinite breakwater problem. Breakwater gap solutions can be built out of semi-infinite solutions. To solve for a gap, it is therefore necessary to make multiple passes through the body of the program with "semi-infinite breakwater" portions of a gap then add the results for the final desired values.

Equations for this program are explained in detail in the text of this report. Due to their length and complexity, they are not repeated

in the summary which follows; but to the maximum extent possible, similar notation is used in the programs and in the text to facilitate cross reference.

Using text notation TH, THN, the program first determines necessary angles, and R. These are then used to find FM, FP and their theta and R derivatives.

Next SIGMA 1 and SIGMA 2 are derived and used in Fresnel integrals to determine C and S. These are then used to give U1, W1, U2, W2 and the derivatives of each with respect to both Theta and R.

At this point, one has all the input needed to find A, B, and their Theta and R derivatives.

Continuing the building process, the derivatives of A and B are used to find $\partial A/\partial X$, $\partial A/\partial Z$, $\partial B/\partial X$, and $\partial B/\partial Z$.

If the solution is only desired for a semi-infinite breakwater, these values are used directly to find the coefficient of diffraction and the wave direction of travel.

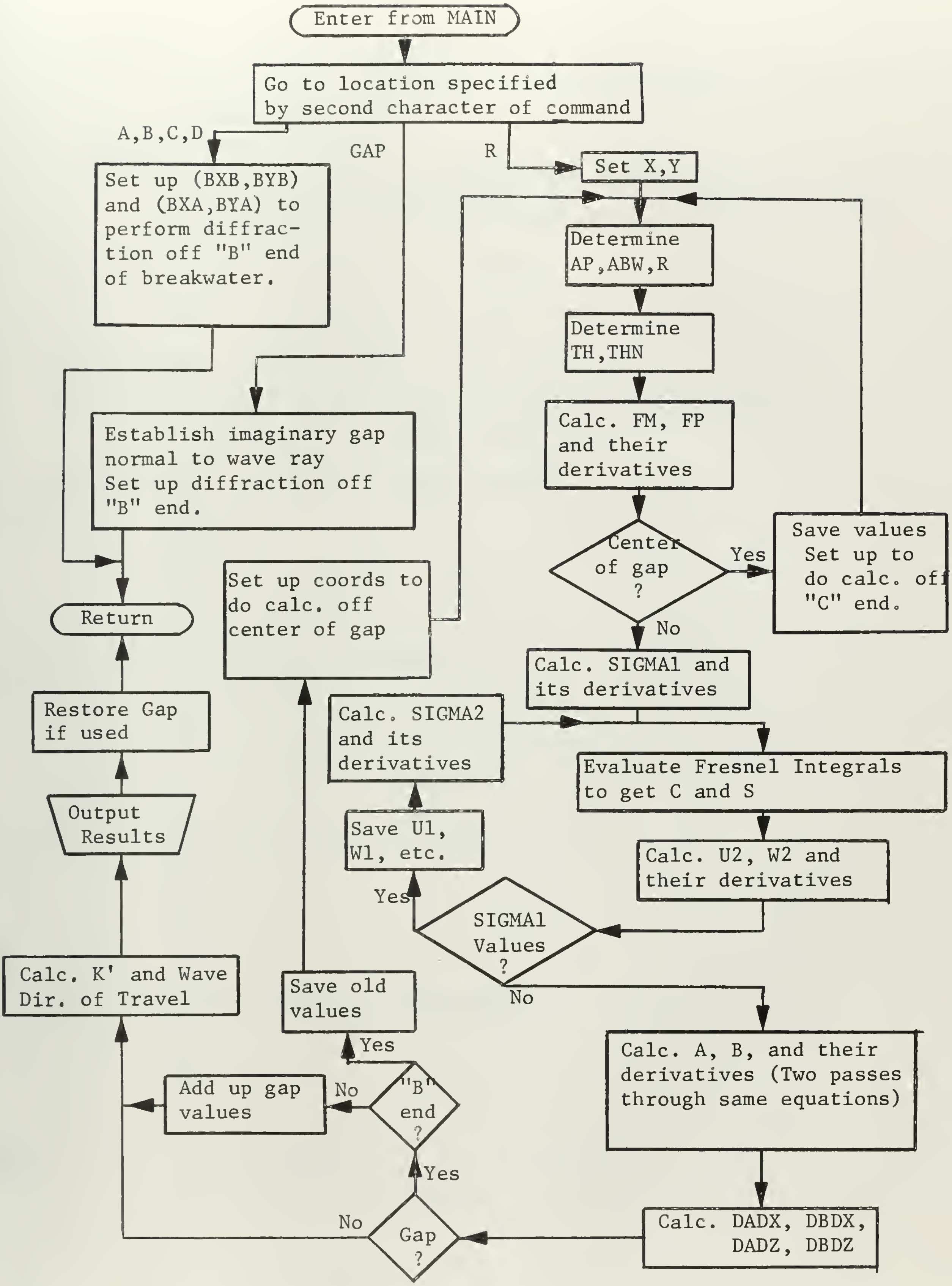
If a gap solution is required, A, B, and their derivatives are saved as a partial solution and coordinates (BXB, BYB) are established to coincide with the "center" of the breakwater gap. It is then necessary to go back and repeat those steps necessary to solve for FP and its derivatives. This information is used to determine exponential derivatives needed for the solution.

Coordinates (BXB, BYB), (BXA, BYA) are then changed to coincide with the C-D breakwater. All steps are repeated as required to find A, B, and all their X, Z derivatives again.

It is then possible to add all partial solutions for the gap to give totals for A, B, and their derivatives. These are used to find the coefficient of diffraction and the wave direction of travel. Upon

completion of a gap solution, coordinates of the imaginary breakwater are restored so that further calculations can be made if desired.

MACRO-FLOW CHART OF RSDIFF



```
      SUBROUTINE RSDIFF(LL,IWD2,F1,F2)
C     PROGRAM TO CALCULATE COEFFICIENT OF DIFFRACTION AND DIRECTION OF
C              WAVE (FRONT) TRAVEL AT END OF BREAKWATERS OR AT A
C              BREAKWATER GAP.
      DIMENSION DMAT(50,40)
      COMMON DMAT,IXN,IYN,PI,G,GRID,X,Y,DXY,D1X,D1Y,D2X,D2Y,D2XY,
     1       A,AD,T,FK,BN,B1,B2,C,PK,PPK,CPPK,CN,DDDC,DCDX,DCDY,
     2        S,NBW,BXAS,BYAS,BXBS,BYBS,BXCS,BYCS,BXDS,BYDS,WLD
C
C     GO TO LOCATION SPECIFIED BY COMMAND .
C
      GO TO (5001,5002,5003,5004,5005,5006,5006),LL
C
C     BOOKKEEPING TO SET UP DATA FOR CALCULATIONS
C     PROGRAM CALCULATES VALUES OFF B END OF BREAKWATER
C     FOR SOLVING GAP, THREE PASSES WILL BE NEEDED TO SOLVE
C              FOR THREE IMAGINARY ENDS .
C
C     DIFFRACTION AT A END OF BREAKWATER
 5001 BXB = BXAS
      BXA = BXBS
      BYB = BYAS
      BYA = BYBS
      NGAP = 1
      GO TO 5008
C DIFFRACTION AT B END OF BREAKWATER
 5002 BXB = BXBS
      BXA = BXAS
      BYB = BYBS
      BYA = BYAS
      NGAP = 1
      GO TO 5008
C     DIFFRACTION AT C END OF BREAKWATER
 5003 BXB = BXCS
      BYB = BYCS
      BXA = BXDS
      BYA = BYDS
      NGAP = 1
      GO TO 5008
C     DIFFRACTION AT D END OF BREAKWATER
 5004 BXB = BXDS
      BYB = BYDS
      BXA = BXCS
      BYA = BYCS
      NGAP = 1
 5008 WRITE (7,5099) IWD2
 5099 FORMAT (30H DIFFRACTION CALCULATIONS FOR   , A1,
     1       19H END OF BREAKWATER.  )
      RETURN
C
C     DIFFRACTION IN BREAKWATER GAP
```

```
C    ESTABLISH AN IMAGINARY GAP.
C
C
 5005 NGAP = 2
      NN = 1
 5009 DX = BXCS - BXBS
      DY = BYCS - BYBS
      DG = (DX**2 + DY**2)**0.5
      XC = (BXBS + BXCS)/2.
      YC = (BYBS + BYCS)/2.
      N = 3
      GO TO 503
 5007 DT = DG/2. * SIN(A-ABW)
      DXT = DT * SIN(A)
      DYT = DT * COS(A)
      BXB = XC - DXT
      BXA = XC - 3.*DXT
      BXC = XC + DXT
      BXD = XC + 3.*DXT
      BYB = YC + DYT
      BYA = YC + 3.*DYT
      BYC = YC - DYT
      BYD = YC - 3.*DYT
      GO TO (5015,5016),NN
 5015 WRITE (7,5098)
 5098 FORMAT (46 H DIFFRACTION CALCULATIONS FOR BREAKWATER GAP.   )
      WRITE (7,5097)
 5097 FORMAT (52H  BASED ON IMAGINARY GAP ASSUMED MORMAL TO WAVE RAY.)
      WRITE (7,5999) BXB,BYB,BXC,BYC
 5999 FORMAT (21H  IMAGINARY GAP FROM   ,F6.3,1H,,F6.3,
     1       4H TO   ,F6.3,1H,,F6.3)
 5016 RETURN
C
C    SET COORD POINTS
C
 5006 X = F1
      Y = F2
      NGAPS = NGAP
C
C    DETERMINE BEARING AND DISTANCE FROM BWB TO X,Y  (AP,R)
C
  500 N = 1
      DY = Y - BYB
      DX = X- BXB
      R = (DX**2 + DY**2)**0.5 * GRID + 0.0000001
  503 IF (ABS(DX)-0.00001) 5011,5012,5012
 5011 ABW = PI/2.
      GO TO 5014
 5012 ABW = ATAN(DY/DX)
C    QUADRANT CHECK
 5014 IF (DY*DX) 504,504,506
```

```
  504 IF(DX) 507,505,508
  505 ABW = PI/2.
  506 IF(DY) 507,509,509
  507 ABW = ABW + PI
      GO TO 509
  508 ABW = ABW + 2.*PI
      IF (ABW-2.*PI) 509,502,502
  502 ABW = ABW - 2.*PI
  509 GO TO (511,513,5007),N
C
C   DETERMINE BREAKWATER BEARING  (ABW)
C
  511 AP = ABW
      DY = BYA - BYB
      DX = BXA - BXB
      N = 2
      GO TO 503
C
C   DETERMINE ANGLE FROM BW TO RAY(THN)
C
  513 THN = ABS(A-ABW)
C
C   DETERMINE ANGLE FROM BW TO X,Y (TH)
C
      TH = ABS(AP-ABW)
      IF (TH-PI) 516,516,515
  515 TH = 2.*PI - TH
  516 IF (THN-PI) 518,518,517
  517 THN = 2.*PI - THN
C   SET MISC SUMMATIONS TO BE USED LATER
  518 FLTK = 2.*PI/(C*T)
      ZZ = (FLTK*R/PI)**0.5
      CTM = COS (TH-THN)
      CTP = COS (TH+THN)
C
C   CALC FM, FP, AND THEIR DERIVATIVES.
C
      FM = FLTK*R*CTM
      FP = FLTK*R*CTP
      DFMDR = FLTK*CTM
      DFPDR = FLTK*CTP
      DFMDTH = -FLTK*R*SIN (TH-THN)
      DFPDTH = -FLTK*R*SIN (TH+THN)
C   NGAP = 1....NO BW,,,  NGAP = 2....DOING LEFT SIDE
C   NGAP = 3 ....DOING CENTER,,,,  NGAP = 4 ....DOING RIGHT SIDE
      GO TO (519,519,577,519),NGAP
C
C   USE SIGMA VALUES TO CALC U, W, AND THEIR DERIVATIVES
C         THROUGH THE USE OF FRESNEL INTEGRALS.
C   CALC SIG1 VALUES FIRST....THEN SIG2 VALUES  (USE LOOP)
C
```

```
  519 SIG2 = 2.*ZZ*SIN (0.5*(TH-THN))
      THX = 0.5*(TH-THN)
      IF (SIG2) 5191,5191,5192
 5191 NQ = 1
      GO TO 5193
 5192 NQ = 2
 5193 M = 1
  510 FT = PI*SIG2**2/2.
      IF (ABS(THX) - 0.000001) 5101,5102,5102
 5101 DSIG2T = ZZ
      GO TO (5104,5105),M
 5105 DSIG2T = -ZZ
      GO TO 5104
 5102 DSIG2T = SIG2*COS(THX)/(2.*SIN(THX))
 5104 SI = ABS(SIG2)
C
C   EVALUATE FRESNEL INTEGRALS (CC AND SS)
C
      IF (SI-3.) 512,540,540
C   IF SI L.T. 3. USE SERIES SOLUTION
  512 SI = SI*((PI/2.)**0.5)
      SI4 = SI**4
      FN = 1.
      TERM = SI
      CC = SI
  514 FN = FN+1.
      TERM = - TERM*SI4*(4.*FN-7.)/(16.*FN**3-52.*FN**2+54.*FN-18.)
      CC = CC+TERM
      IF (ABS (TERM) - 0.0001) 524,514,514
  524 CC = CC*((2./PI)**0.5)
      FN = 1.
      SS = (SI**3)/3.
      TERM = SS
  530 FN = FN+1.
      TERM = -TERM*SI4*(4.*FN-5.)/(16.*FN**3-28.*FN**2+14.*FN-2.)
      SS = SS+TERM
      IF (ABS (TERM)-0.0001) 538,530,530
  538 SS = SS*((2./PI)**0.5)
      GO TO 545
C   IF SI G.T. 3. USE SIN/COS APPROXIMATIONS
  540 Z2 = (PI/2.)*(SI**2-2./(PI*SI)**2)
      CC = 0.5 + SIN (Z2)/(PI*SI)
      SS = 0.5 - COS (Z2)/(PI*SI)
C
C   CALC U2,W2,AND ALL THEIR DERIVATIVES.
C
  545 U2 = 0.5*(1.-SS-CC)
      W2 = 0.5*(SS-CC)
      IF (ABS(SIG2)-0.000001) 5451,5452,5452
 5452 DSIG2R = (SIG2/(2.*R))*(SIG2/(ABS(SIG2)))
      DSIG2T = DSIG2T * (SIG2/(ABS(SIG2)))
```

```
      GO TO 5453
 5451 DSIG2R = SIG2/(2.*R)
 5453 DU2DR = -0.5*(SIN(FT)*DSIG2R + COS(FT)*DSIG2R)
      DW2DR =0.5*(SIN (FT)*DSIG2R-COS (FT)*DSIG2R)
      DU2DTH = -0.5*(SIN (FT)*DSIG2T + COS (FT)*DSIG2T)
      DW2DTH = 0.5*(SIN (FT)*DSIG2T - COS (FT)*DSIG2T)
C
C   ON FIRST PASS RESET VALUES TO SHOW U1,W1,AND THEIR DERIVATIVES
C   SET SIGMA2 AND GO BACK TO CALCULATE U2,W2,AND THEIR DERIVATIVES.
C
      GO TO (550,560),M
  550 U1 = U2
      W1 = W2
      DU1DR = DU2DR
      DW1DR = DW2DR
      DU1DTH = DU2DTH
      DW1DTH = DW2DTH
      SIG2 = -2.*ZZ*SIN (0.5*(TH+THN))
      THX = 0.5*(TH+THN)
      M = 2
      GO TO 510
C
C   USE SAME BASIC A AND B EQUATIONS TO CALCULATE BOTH R AND THETA
C          DERIVATIVES OF A AND B.
C   FIRST DO R DEVIVATIVES.
C
  560 M=1
      DFM = DFMDR
      DFP = DFPDR
      DU1 = DU1DR
      DU2 = DU2DR
      DW1 = DW1DR
      DW2 = DW2DR
      A1 = U1 * COS(FM) + W1 * SIN(FM)
      A2 = U2 *COS(FP) + W2 *SIN(FP)
      B1X = W1 *COS(FM) - U1 * SIN(FM)
      B2X = W2 * COS(FP) - U2 * SIN(FP)
  565 DA1 = DU1 *COS(FM) - U1*SIN(FM)*DFM + DW1*SIN(FM)
     1          + W1*COS(FM)*DFM
      DA2 = DU2 * COS(FP) - U2*SIN(FP)*DFP + DW2 * SIN(FP)
     1        + W2*COS(FP)*DFP
      DB1 = DW1 * COS(FM) - W1*SIN(FM)*DFM - DU1*SIN(FM)
     1         - U1*COS(FM)*DFM
      DB2 = DW2*COS(FP) - W2*SIN(FP)*DFP - DU2*SIN(FP)
     1          - U2*COS(FP)*DFP
C   WATCH OUT FOR PROPER QUADRANT
      GO TO (570,580),NQ
  570 DADTH = DA1 + DA2
      DBDTH = DB1 + DB2
      GO TO (572,575),M
  572 AA = A1 + A2
```

```
      BB = B1X + B2X
      GO TO 585
  580 IF(NGAP-1) 5801,5801,5811
 5801 DADTH = -SIN(FM)*DFM - DA1 + DA2
      DBDTH = -COS(FM)*DFM - DB1 + DB2
      GO TO 5812
 5811 DADTH = DA2 -DA1
      DBDTH = DB2 -DB1
 5812 GO TO (582,575),M
  582 IF(NGAP-1) 5821,5821,5822
 5821 AA = COS(FM) - A1 + A2
      BB = -SIN(FM) - B1X+ B2X
      GO TO 585
 5822 AA = -A1 + A2
      BB = -B1X + B2X
C
C   SAVE R VALUES....SET VARIABLES SO AS TO CALCULATE THETA
C        DERIVATIVES AND DO IT OVER AGAIN.
C
  585 M = 2
      DADR = DADTH
      DBDR = DBDTH
      DFM = DFMDTH
      DFP = DFPDTH
      DU1 = DU1DTH
      DU2 = DU2DTH
      DW1 = DW1DTH
      DW2 = DW2DTH
      GO TO 565
C
C   CALC DERIVATIVES OF A AND B WITH REGARDS TO X AND Z.
C
  575 DADX = DADR*SIN (TH) + DADTH*COS (TH) / R
      DBDX = DBDR*SIN (TH) + DBDTH*COS (TH)/R
C   WATCH OUT FOR QUADRANTS
 5753 GO TO (5751,5752,5751,5751),NGAP
 5752 DADZ = -DADR*COS(TH) + DADTH*SIN(TH)/R
      DBDZ = -DBDR*COS(TH) + DBDTH*SIN(TH)/R
      GO TO 5754
 5751 DADZ = DADR * COS(TH) - DADTH*SIN(TH)/R
      DBDZ = DBDR*COS (TH)-DBDTH*SIN (TH)/R
C
C   IF THIS IS A GAP GP BACK AND DO AGAIN FOR CENTER AND RIGHT SIDE
C
 5754 GO TO (574,576,577,578),NGAP
C
C   SAVE OLD VALUES FOR LEFT SIDE OF BW
C
  576 DADZ1 = DADZ
      DBDZ1 = DBDZ
      DADX1 = DADX
```

```
      DBDX1 = DBDX
      TH11 = TH
      AA1 = AA
      BB1 = BB
C   DO CENTER NEXT
      NGAP = 3
      BYA = BYB
      BXA = BXB
      BXB = (BXB+BXC)/2.
      BYB = (BYB+BYC)/2.
      GO TO 500
C
C   SAVE CENTER VALUES (ONLY A PARTIAL SET)
C
  577 FACFP = COS(FP)
      FBCFP = -SIN(FP)
      FF1 = DFPDR*SIN(TH) + DFPDTH * COS(TH)/R
      FF2 = DFPDR*COS(TH) - DFPDTH*SIN(TH)/R
      FACFPX = -SIN(FP)*FF1
      FBCFPX = COS(FP)*FF1
      FACFPZ = SIN(FP)*FF2
      FBCFPZ = -COS(FP)*FF2
C   GO BACK AND DO RT SIDE OF BW
      BXA = BXD
      BYA = BYD
      BXB = BXC
      BYB = BYC
      NGAP = 4
      GO TO 500
C
C   IF THIS WERE A GAP PROPLEM MAKE SUMMATIONS.
C
C   DOES POINT LAY BETWEEN GAP TIPS OR BEHIND A BREAKWATER.
C   MAKE BW SUMMATIONS
  578 IF ((TH11 + A - PI)*(TH - A)) 579,579,581
  581 DADX = DADX + FACFPX
      DBDX = DBDX + FBCFPX
      DADZ = DADZ + FACFPZ
      DBDZ = DBDZ + FBCFPZ
      AA = AA + FACFP
      BB = BB + FBCFP
  579 DADX = DADX1 + DADX
      DADZ = DADZ1 + DADZ
      DBDX = DBDX1 + DBDX
      DBDZ = DBDZ1 + DBDZ
      AA = AA1 + AA
      BB = BB1 + BB
C
C   MADE IT.......WHEW........
C   NOW CALC DIR. OF TRAVEL AND COEFF. OF DIFFRACTION
C
```

```
  574 IF (ABS(AA*DBDZ-BB*DADZ)-0.00001) 5741,5742,5742
 5741 DIR = PI/2.
      GO TO 5744
 5742 DIR = ATAN((AA*DBDX-BB*DADX)/(AA*DBDZ-BB*DADZ))
C    DIR IS WRT BW.....RETURN TO X,Y COORD SYSTEM
C    CORRECT FOR NEGATIVE ATANS IN SECOND QUADRANT.
 5734 IF (DIR) 5743,5744,5744
 5743 DIR = DIR + PI
 5744 ABW1 = ABW
      IF (ABS(A-ABW1)-PI) 5748,5748,5749
 5749 ABW1 = ABW1-2.*PI
 5748 IF ((2.*PI-(A-ABW1))*(A-ABW1))5746,5746,5745
 5745 DIR = DIR + ABW
      GO TO 5747
 5746 DIR = ABW - DIR
 5747 IF (DIR-2.*PI) 5731,5732,5732
 5732 DIR = DIR-2.*PI
 5731 DIRD = DIR*180./PI
      CDIF = (AA**2 + BB**2)**0.5
C
C    PRINT OUT RESULTS
C
      WRITE (7,5096)
      WRITE (7,5095) AD,T,C
      WRITE (7,5094) X,Y
      WRITE (7,5093) DIRD,CDIF
      NGAP = NGAPS
C
C    RESTORE ARTIFICIAL GAP IF USED
C
      NN = 2
      IF (NGAP-2) 5016,5009,5009
 5096 FORMAT (30H           WAVE CHARACTERISTICS  )
 5095 FORMAT (9H   ANGLE   ,F7.2, 18H DEG.,     PERIOD   ,F6.2,
    1      16H SEC.,      SPEED, F5.2, 9H FT/SEC.  )
 5094 FORMAT (14H  AT POINT X = , F5.2, 5H  Y= , F5.2)
 5093 FORMAT (27H           DIR OF TRAVEL IS   ,F7.2,
    1        29H DEG.,COEF OF DIFFRACTION IS      , F7.4)
      RETURN
      END
```

7. SUBROUTINE RSSHOR

Subroutine which calculates shoreline location from a depth array.

SUBROUTINE NAME: RSSHOR

<u>Variables used in Program</u>:

COMMON variables are listed under MAIN.

D . . . . Fraction of distance between two grid points at which shoreline crossing has been interpolated.

DX, DY . . . . D distance along X or Y grid line respectively.

INC . . . . . . Index used to keep track of which of 6 possible grid crossings is being checked; INC=1 during original search for a shoreline segment.

IX, IY . . . . Coordinates at primary point of shoreline search; program looks for shoreline between (IX, IY) and (IX + 1, IY) or between (IX, IY) and (IX, IY + 1).

IX2, IY2 . . . Coordinates at secondary grid point opposite (IX, IY). Shoreline check occurs between two points.

NCK(LL) . . . . A check list of coordinate locations at which grid crossings have already been found. Coordinates are stored as an integer (100 * IX + IY); with a plus value indicating grid crossing between (IX, IY) and (IX + 1, IY), and a negative value indicating grid crossing was between (IX, IY) and (IX, IY + 1).

NN	Do loop counter, no significance.
NNS	Index; NNS=1 if last grid crossing was along Y grid line; NNS=2 if last grid crossing was along X grid line.
NS	Index; NS=+1 if searching between IX and IX +1; NS= -1 if searching between IY and IY +1.
NSL	Index of total number of grid crossing located and stored in NCK(--).
XD, YD	Coordinates of shoreline at a grid crossing point.
XS, YS	First coordinate points along a shoreline segment; used to close loop if shoreline traces back to origin as on an island or enclosed bay.

Summary of Program

When called by MAIN, RSSHOR finds and traces shoreline segments across a DMAT array by using the fact that, only at a shoreline, the product of two adjacent depths (DMAT (IX, IY)) * (DMAT (IX2, IY2)) is negative.

Starting at (2,2), the product of adjacent grid depths is determined along grid lines parallel to the Y axis until an initial starting point of a shoreline segment is found. Once a shoreline is found, it is then traced to its ending, either at the matrix edge or back at its start such as at an island.

Each time a shoreline grid crossing is found, the coordinates of (IX, IY) are stored as integer (100 * IX + IY) in NCK (NSL). It is given a negative value if the grid crossing were between (IX, IY) and (IX + 1, IY); it is given a positive value if the crossing were between

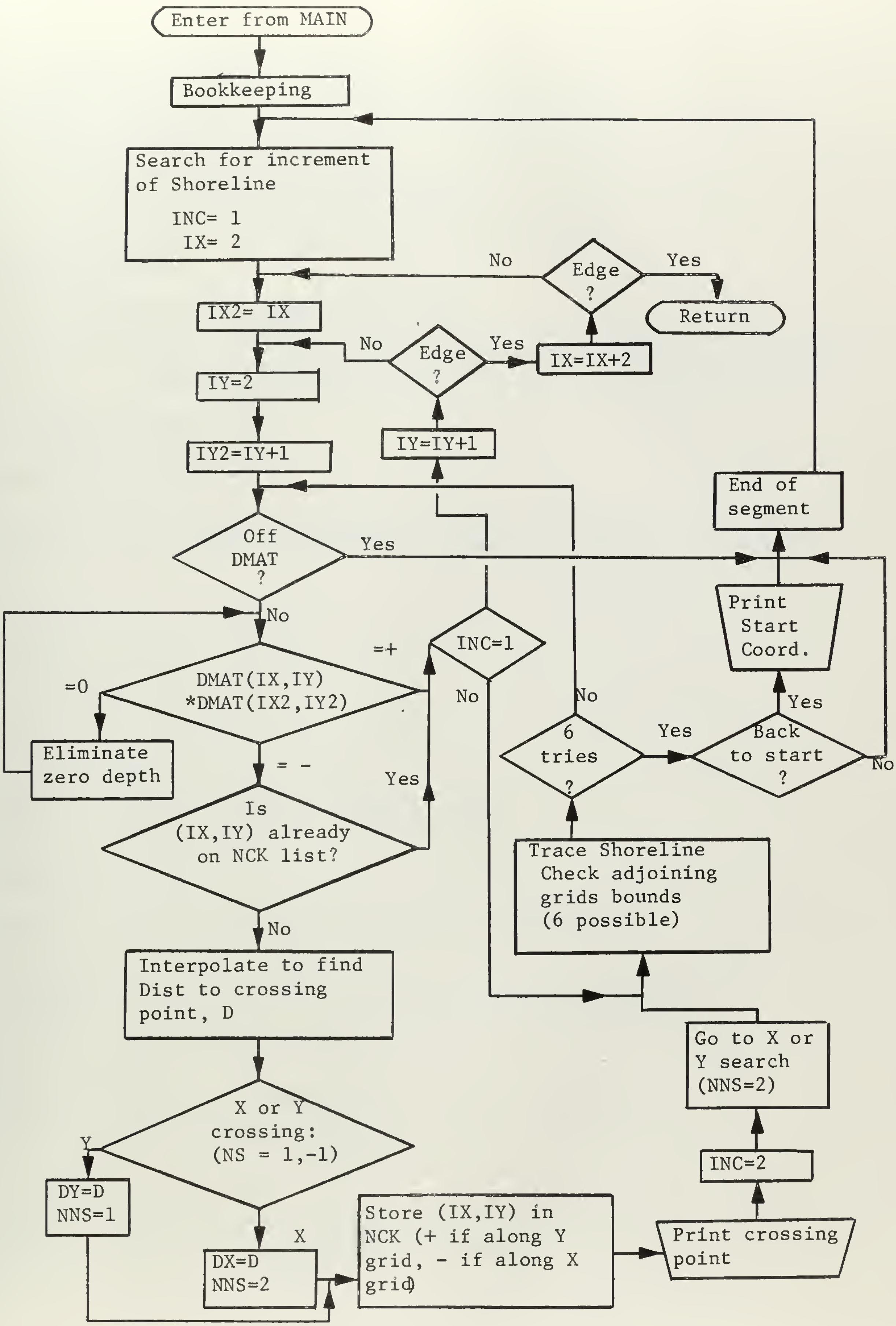
(IX, IY) and (IX, IY + 1). Each possible crossing point is first checked against NCK to be sure that it is in fact a new crossing point, not a duplicate.

Shoreline traces are made by checking the six adjacent grid bounds for a new crossing; when found, it is stored in NCK. A check is then made of its 6 adjacent grid bounds. The NCK listing eliminates any line backing up upon itself.

Each time a shoreline is ended, a new search starts at (1,2) until all segments have been found.

Output is a series listing of consecutive shoreline coordinates.

MACRO-FLOW CHART OF RSSHOR



```
      SUBROUTINE RSSHOR (DMMY)
C   SUBROUTINE WHICH WILL LOCATE THE SHORELINE
C   BASIC APPROACH...PRODUCT OF ADJACENT DEPTHS IS NEGATIVE
C      WHEN CROSSING SHORELINE)
C   USE STRAIGHT LINE INTERPOLATION BETWEEN GRID POINTS OF APPOSITE
C      SIGN TO APPROXIMATE SHORELINE
C   USE NCK (XXX) AS CHECK LIST FOR COORDINATE LOCATIONS
C
C   BOOKKEEPING
C
      DIMENSION DMAT(50,40),NCK(200)
      COMMON DMAT,IXN,IYN,PI,G,GRID,X,Y,DXY,D1X,D1Y,D2X,D2Y,D2XY,
     1      A,AD,T,FK,BN,B1,B2,C,PK,PPK,CPPK,CN,DDDC,DCDX,DCDY,
     2       S,NBW,BXAS,BYAS,BXBS,BYBS,BXCS,BYCS,BXDS,BYDS,WLD
      NSL = 1
C   ZERO OUT CHECK LIST
      DO 402 LL = 1,200
  402 NCK(LL) = 0
C
C   SEARCH FOR INCREMENT OF SHORELINE.
C   INC = 1  ..  NOT IN PROCESS OF TRACING SHORELINE SEGMENT
C
  404 INC = 1
      IX = 1
  406 NS = 1
      IX2 = IX
      IY = 2
  410 IY2 = IY+1
C
C   EDGE CHECK
C
  420 IF ((IX-1)*(IXN-IX)) 490,422,422
  422 IF ((IY-1)*(IYN-IY)) 490,425,425
C
C  CHECK PRODUCT
C
  425 IF (DMAT(IX,IY)*DMAT(IX2,IY2)) 430,426,411
C
C  INCREMENT TO NEXT POINT
C   5EEP INCREMENTING UNTIL ENTIRE GRID IS CHECKED...
C
  411 IF (INC-1) 460,412,460
  412 IY = IY+1
      IF (IYN-IY) 414,414,410
  414 IX = IX + 2
      IF (IXN-IX) 495,406,406
C
C   ELIMINATE ANY ZERO DEPTHS
C
  426 IF (DMAT(IX,IY)) 428,427,428
  427 DMAT(IX,IY) = DMAT(IX,IY) + 0.0001
```

```
  428 IF (DMAT(IX2,IY2)) 425,429,425
  429 DMAT(IX2,IY2) = DMAT(IX2,IY2) + 0.0001
      GO TO 425
C
C    CHECK FILE FOR PREVIOUS LISTING OF POINT
C
  430 DO 445 NN = 1,NSL
      IF (NCK(NN) - NS*(100*IX+IY)) 445,411,445
  445 CONTINUE
C
C    INTERPOLATE BETWEEN GRID POINTS
C    WAS IT AN X OR Y CROSSING...
C
      D = DMAT(IX,IY)/(DMAT(IX,IY) - DMAT(IX2,IY2))
      IF (NS-1) 450,452,452
  450 DX = D
      DY = 0.
      NNS = 2
      GO TO 455
  452 DY = D
      DX = 0.
      NNS = 1
C
C    STORE GRID POINT IN FILE
C    STORE + VALUE IF ALONG Y GRID
C    STORE - VALUE IF ALONG X GRID
C
  455 NCK(NSL) = NS*(100*IX + IY)
      XD = FLOAT(IX) + DX
      YD = FLOAT(IY) + DY
      IF (INC-1) 457,456,457
  456 XS = XD
      YS = YD
      WRITE (7,496)
  496 FORMAT (5X,22H SHORELINE COORDINATES)
C
C    PRINT CROSSING POINT
C
  457 WRITE (7,497) NSL,XD,YD
  497 FORMAT (I5,17X,2F8.3)
      NSL = NSL+1
C    INC = 2 WHEN FOLLOWING SHORELINE
      INC = 2
C
C    SEARCH FOR ADJOINING SHORELINE GRID CROSSING
C    LOOK IN 6 POSSIBLE DIRECTIONS
C
C
  460 GO TO (459,470),NNS
  459 GO TO (462,462,463,464,465,466,467,468), INC
  462 IX = IX+1
```

```
      INC = 3
  461 IX2 = IX
      NS = 1
      GO TO 420
  463 IX = IX-2
      IF (IX) 4652,4652,4631
 4631 INC = 4
      GO TO 461
  464 INC = 5
      IY2 = IY
  469 IX2 = IX+1
  458 NS = -1
      GO TO 420
 4652 IY2 = IY
  465 INC = 6
      IX = IX+1
      GO TO 469
  466 INC = 7
      IY = IY+1
      IY2 = IY
      GO TO 458
  467 INC = 8
      IX = IX-1
      IF (IX) 468,468,469
C
C   DID WE GO BACK TO STARTING POINT...
C
  468 IF (ABS(XD-XS)-2.) 481,481,490
  481 IF (ABS(YD-YS)-2.) 482,482,490
C   IF BACK AT START, PRINT STARTING COORD.
  482 WRITE (7,497) NSL,XS,YS
      GO TO 490
  470 GO TO (472,472,473,474,475,476,477,468), INC
  472 IY = IY+1
      INC = 3
  471 IY2 = IY
      NS = -1
      GO TO 420
  473 IY = IY-2
      INC = 4
      GO TO 471
  474 INC = 5
      IX2 = IX
  479 IY2 = IY+1
  480 NS = 1
      GO TO 420
  475 INC = 6
      IY = IY+1
      GO TO 479
  476 INC = 7
      IX = IX+1
```

```
      IX2 = IX
      GO TO 480
  477 INC = 8
      IY = IY-1
      GO TO 479
C
C   END OF SHORELINE SEGMENT
C
  490 WRITE (7,499)
  499 FORMAT (5X, 22H END SHORELINE SEGMENT   )
C   GO LOOK FOR ANOTHER SEGMENT
      GO TO 404
  495 RETURN
      END
```

APPENDIX (C)

FORMAT FOR COMMAND/DATA INPUT

CARD COLUMNS

1----20	21-25	26-30	31-40	41-50	51-60	61-70
ANALYTICAL MATRIX	IX1	IX2	DX	DM		
	IY1	IY2	DY			
LIMITS OF TOPO	IX1	IX2				
	IY1	IY2				
TOPO IN FEET FOLLOWS						
TOPO IN FATH FOLLOWS						
INCREMENT			S			
RAY DATA 1	NTC		T	AD	X	Y
RAY DATA 2			B1	BN		
WATER LEVEL			F1			
TRACE RAY						
LOCATE SHORELINE						
BREAKWATER			BXAS	BYAS	BXBS	BYBS
BREAKWATER			BXCS	BYCS	BXDS	BYDS
GAP						
SIZE GRID	IXN	IYN	GRID			
WAVE LENGHTS DIFFR			WLD			
LENGTH DIFFRACTED			WLD			
ELIMINATE BRKWTRS						
DIFFRACT A						
DIFFRACT B						
DIFFRACT C						
DIFFRACT D						
DIFFRACT GAP						
DIFFR COORD			X	Y		

(CONTINUED)

CARD COLUMNS

1----20	21-25	26-30	31-40	41-50	51-60	61-70
WAVE SPEED			C			
CALC WAVE SPEED			X	Y		
(BLANK)	N1	N2	F1	F2	F3	F4
ALL DONE						

NOTE...RIGHT JUSTIFY INTEGER DATA ITEMS

APPENDIX (D)

REFRACTION TEST ON ANALYTICAL BEACH
(SEE TABLE 1.)

ANALYTICAL DATA FROM X= 1 TO 50 AND Y= 1 TO 40
TEST CASE 1 PERIOD = 4.0SEC. ANGLE= 45.0DEG.X= 3.00Y= 3.00

N	X	Y	A	C	BETA	COEF RFTN	SHOAL COEF	CURV	S	DPTH
1	3.000	3.000	45.000	20.49	1.000	1.000	1.00	0.0001	4.00	56.00
2	5.828	5.829	45.019	20.48	1.000	1.000	1.00	0.0002	4.00	50.34
3	7.241	7.244	45.040	20.47	1.001	1.000	1.00	0.0003	2.00	47.51
4	8.654	8.659	45.074	20.46	1.001	0.999	1.00	0.0004	2.00	44.68
5	10.066	10.076	45.125	20.43	1.002	0.999	0.99	0.0007	2.00	41.85
6	11.476	11.495	45.222	20.40	1.004	0.998	0.99	0.0010	2.00	39.01
7	12.883	12.916	45.369	20.35	1.006	0.997	0.98	0.0016	2.00	36.17
8	14.285	14.342	45.589	20.27	1.010	0.995	0.98	0.0023	2.00	33.32
9	15.681	15.775	45.917	20.15	1.016	0.992	0.97	0.0035	2.00	30.45
10	17.066	17.217	46.402	19.97	1.024	0.988	0.96	0.0051	2.00	27.57
11	17.753	17.943	46.728	19.85	1.029	0.986	0.96	0.0061	1.00	26.11
12	18.436	18.674	47.115	19.71	1.036	0.983	0.95	0.0074	1.00	24.65
13	19.114	19.409	47.579	19.54	1.043	0.979	0.94	0.0089	1.00	23.18
14	19.785	20.151	48.134	19.33	1.052	0.975	0.94	0.0106	1.00	21.70
15	20.118	20.524	48.453	19.21	1.058	0.972	0.94	0.0115	0.50	20.95
16	20.448	20.899	48.798	19.08	1.063	0.970	0.93	0.0126	0.50	20.20
17	20.776	21.277	49.175	18.93	1.069	0.967	0.93	0.0137	0.50	19.45
18	21.102	21.656	49.586	18.78	1.076	0.964	0.93	0.0150	0.50	18.69
19	21.424	22.038	50.033	18.60	1.083	0.961	0.92	0.0163	0.50	17.92
20	21.744	22.423	50.520	18.41	1.091	0.957	0.92	0.0177	0.50	17.15
21	22.060	22.810	51.050	18.21	1.099	0.954	0.92	0.0193	0.50	16.38
22	22.372	23.200	51.627	17.98	1.108	0.950	0.92	0.0210	0.50	15.60
23	22.681	23.594	52.255	17.73	1.117	0.946	0.92	0.0229	0.50	14.81
24	22.984	23.991	52.939	17.45	1.128	0.942	0.91	0.0249	0.50	14.02
25	23.283	24.392	53.683	17.15	1.139	0.937	0.91	0.0271	0.50	13.22
26	23.576	24.797	54.495	16.82	1.150	0.932	0.91	0.0296	0.50	12.41
27	23.864	25.206	55.379	16.45	1.163	0.927	0.92	0.0323	0.50	11.59
28	24.144	25.620	56.346	16.05	1.176	0.922	0.92	0.0353	0.50	10.76
29	24.417	26.039	57.403	15.60	1.191	0.916	0.92	0.0386	0.50	9.92
30	24.682	26.463	58.563	15.11	1.206	0.911	0.93	0.0424	0.50	9.07
31	24.938	26.892	59.841	14.55	1.222	0.905	0.93	0.0468	0.50	8.22
32	25.184	27.328	61.253	13.93	1.239	0.898	0.94	0.0519	0.50	7.34
33	25.419	27.769	62.824	13.23	1.257	0.892	0.96	0.0579	0.50	6.46
34	25.640	28.218	64.586	12.43	1.276	0.885	0.98	0.0653	0.50	5.56
35	25.847	28.673	66.585	11.51	1.297	0.878	1.00	0.0745	0.50	4.65
36	26.036	29.136	68.886	10.43	1.318	0.871	1.04	0.0869	0.50	3.73
37	26.205	29.606	71.610	9.14	1.341	0.864	1.10	0.1048	0.50	2.79
38	26.349	30.085	74.994	7.50	1.365	0.856	1.20	0.1347	0.50	1.83
39	26.458	30.573	79.709	5.19	1.390	0.848	1.42	0.2045	0.50	0.85

RAY STOPPED

APPENDIX (E)

SAMPLE PROGRAM RUN

DIFFRACTION OPERATIONS CALCULATE VALUES FOR AN ASSORTMENT OF MISCELLANEOUS CASES.

REFRACTION OPERATIONS SHOW VIRGINIA BEACH TESTS (SEE FIG. 4)

INPUT DATA DECK.........

WAVE SPEED		20.			
SIZE GRID	50	40 80.			
BW		10.	10.	20.	20.
BW		22.	22.	30.	30.
RAY DATA 1	9	4.	90.	1.	1.
DIFFRACT A					
DIFFR COORD		8.	14.		
		10.	13.		
		12.	14.		
RAY DATA 1	11	4.	135.	1.	1.
DIFFR COORD		6.	12.		
		6.5	12.5		
		7.	13.		
		8.	14.		
		12.	13.		
DIFFRACT D					
DIFFR COORD		25.	30.		
		26.	32.		
		27.	33.		
		27.5	33.5		
		28.	34.		
RAY DATA 1	1	4.0	90.	1.	1.
DIFFRACT GAP					
DIFFR COORD		21.	26.		
		23.	25.		
		20.5	23.		
ELIMIN BREAKWATERS					
BW		10.	10.	20.	10.
DIFFRACT A					
RAY DATA 1	1	4.	90.	2.	2.
DIFFR COORD		9.9	15.		
		10.	15.		
		10.1	15.		
DIFFRACT B					
DIFFR COORD		19.9	15.		
		20.	15.		
		20.1	15.		
ELIMINATE BRKWTRS					

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SIZE GRID               50   40 3000.
ANALYTICAL MATRIX        1   50 0.        -24.
                        33   40 0.
LIMITS OF TOPO           1   50
                         1   32
TOPO IN FATH FOLLOWS
 95    95    96    97    97    98    98    99    96    95
 92    90    88    85    83    82    82    82    82    82
 82    82    82    82    82    81    82    82    82    82
 83    83    84    85    85    86    86    87    87    88
 88    89    98    92    92    93    93    94    93    92
 92    93    94    95    95    96    96    97    96    94
 92    89    87    84    81    80    80    80    81    81
 81    81    81    80    80    80    80    80    81    81
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       (INTERIM CARDS OMITTED)
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  . . . . . . . . . . . . . . . . . . .
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -35   -30   -25   -20   -15
 435   44    445   45    452   455   458   46    465   47
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -35   -30   -25   -20   -15   -10   -00    09
 12    19    22    28    30    33    36    38    40    405
 405   41    415   42    425   435   435   44    445   45
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -40   -35   -30   -25   -20
-15   -10   -00    08    12    19    20    22    25    29
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  . . . . . . . . . . . . . . . . . . .
       (INTERIM CARDS OMITTED)
  . . . . . . . . . . . . . . . . . . .
  . . . . . . . . . . . . . . . . . . .
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -40   -40   -40   -40   -40
-40   -40   -40   -40   -40   -40   -40   -40   -30   -28
RAY DATA 1              31    4.         30.       5.0        11.1
TRACE RAY
RAY DATA 1              32    4.         30.       6.0        9.3
TRACE RAY
RAY DATA 1              33    4.         30.       7.0        7.9
TRACE RAY
LOCATE SHORELINE
ALL DONE
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OUTPUT INFORMATION DECK........

 WAVE SPEED IS SET AT  20.00 FT/S
 GRID IS   50 BY   40 WITH    80.00 FOOT SQUARES.
 BREAKWATER FROM  10.00,  10.00 TO  20.00,  20.00
 BREAKWATER FROM  22.00,  22.00 TO  30.00,  30.00
 DIFFRACTION CALCULATIONS FOR A END OF BREAKWATER.
         WAVE CHARACTERISTICS
   ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X = 8.00   Y= 14.00
         DIR OF TRAVEL IS   92.37 DEG.,COEF OF DIFFRACTION IS  1.1040
         WAVE CHARACTERISTICS
   ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X =10.00   Y= 13.00
         DIR OF TRAVEL IS   82.60 DEG.,COEF OF DIFFRACTION IS  0.5490
         WAVE CHARACTERISTICS
   ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X =12.00   Y= 14.00
         DIR OF TRAVEL IS   62.67 DEG.,COEF OF DIFFRACTION IS  0.2278
         WAVE CHARACTERISTICS
   ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X = 6.00   Y= 12.00
         DIR OF TRAVEL IS  137.41 DEG.,COEF OF DIFFRACTION IS  1.0758
         WAVE CHARACTERISTICS
   ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X = 6.50   Y= 12.50
         DIR OF TRAVEL IS  134.01 DEG.,COEF OF DIFFRACTION IS  0.8031
         WAVE CHARACTERISTICS
   ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X = 7.00   Y= 13.00
         DIR OF TRAVEL IS  128.80 DEG.,COEF OF DIFFRACTION IS  0.5283
         WAVE CHARACTERISTICS
   ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X = 8.00   Y= 14.00
         DIR OF TRAVEL IS  115.00 DEG.,COEF OF DIFFRACTION IS  0.2466
         WAVE CHARACTERISTICS
   ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X =12.00   Y= 13.00
         DIR OF TRAVEL IS   56.31 DEG.,COEF OF DIFFRACTION IS  0.1203
 DIFFRACTION CALCULATIONS FOR D END OF BREAKWATER.
         WAVE CHARACTERISTICS
   ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X =25.00   Y= 30.00
         DIR OF TRAVEL IS  180.17 DEG.,COEF OF DIFFRACTION IS  0.1309
         WAVE CHARACTERISTICS
   ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
  AT POINT X =26.00   Y= 32.00
         DIR OF TRAVEL IS  155.00 DEG.,COEF OF DIFFRACTION IS  0.2466
         WAVE CHARACTERISTICS
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  ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =27.00   Y= 33.00
        DIR OF TRAVEL IS  141.20 DEG.,COEF OF DIFFRACTION IS  0.5283
        WAVE CHARACTERISTICS
  ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =27.50   Y= 33.50
        DIR OF TRAVEL IS  135.99 DEG.,COEF OF DIFFRACTION IS  0.8031
        WAVE CHARACTERISTICS
  ANGLE  135.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =28.00   Y= 34.00
        DIR OF TRAVEL IS  132.59 DEG.,COEF OF DIFFRACTION IS  1.0758
DIFFRACTION CALCULATIONS FOR BREAKWATER GAP.
 BASED ON IMAGINARY GAP ASSUMED NORMAL TO WAVE RAY.
 IMAGINARY GAP FROM 20.000,21.000 TO 22.000,21.000
        WAVE CHARACTERISTICS
  ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =21.00   Y= 26.00
        DIR OF TRAVEL IS   90.00 DEG.,COEF OF DIFFRACTION IS  0.8493
        WAVE CHARACTERISTICS
  ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =23.00   Y= 25.00
        DIR OF TRAVEL IS   77.82 DEG.,COEF OF DIFFRACTION IS  0.2386
        WAVE CHARACTERISTICS
  ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =20.50   Y= 23.00
        DIR OF TRAVEL IS   94.29 DEG.,COEF OF DIFFRACTION IS  0.8883
ELIMINATED BREAKWATERS
BREAKWATER FROM  10.00,  10.00 TO  20.00,  10.00
DIFFRACTION CALCULATIONS FOR A END OF BREAKWATER.
        WAVE CHARACTERISTICS
  ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X = 9.90   Y= 15.00
        DIR OF TRAVEL IS   84.98 DEG.,COEF OF DIFFRACTION IS  0.5569
        WAVE CHARACTERISTICS
  ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =10.00   Y= 15.00
        DIR OF TRAVEL IS   84.28 DEG.,COEF OF DIFFRACTION IS  0.5260
        WAVE CHARACTERISTICS
  ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =10.10   Y= 15.00
        DIR OF TRAVEL IS   83.56 DEG.,COEF OF DIFFRACTION IS  0.4968
DIFFRACTION CALCULATIONS FOR B END OF BREAKWATER.
        WAVE CHARACTERISTICS
  ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =19.90   Y= 15.00
        DIR OF TRAVEL IS   96.44 DEG.,COEF OF DIFFRACTION IS  0.4968
        WAVE CHARACTERISTICS
  ANGLE   90.00 DEG.,      PERIOD   4.00 SEC.,      SPEED20.00 FT/SEC.
 AT POINT X =20.00   Y= 15.00
        DIR OF TRAVEL IS   95.72 DEG.,COEF OF DIFFRACTION IS  0.5260
        WAVE CHARACTERISTICS
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ANGLE 90.00 DEG., PERIOD 4.00 SEC., SPEED20.00 FT/SEC.
AT POINT X =20.10 Y= 15.00
DIR OF TRAVEL IS 95.02 DEG.,COEF OF DIFFRACTION IS 0.5569
ELIMINATED BREAKWATERS
GRID IS 50 BY 40 WITH 3000.00 FOOT SQUARES.
ANALYTICAL DATA FROM X= 1 TO 50 AND Y= 33 TO 40
TOPO DATA FROM X= 1 TO 50 AND Y= 1 TO 32
TEST CASE 31 PERIOD = 4.0SEC. ANGLE= 30.0DEG.X= 5.00Y=11.10

N	X	Y	A	C	BETA	COEF RFTN	SHOAL COEF	CURV	S	DPTH
1	5.000	11.100	30.000	20.42	1.000	1.000	1.00	0.0014	4.00	40.64
2	6.731	12.102	30.159	20.43	1.009	0.995	1.00	0.0012	2.00	41.82
3	8.459	13.109	30.291	20.44	1.024	0.988	1.00	0.0009	2.00	42.10
4	10.185	14.119	30.392	20.43	1.045	0.978	1.00	0.0017	2.00	41.25
5	11.908	15.135	30.622	20.39	1.076	0.964	1.00	0.0023	2.00	38.54
6	13.626	16.158	30.923	20.36	1.112	0.948	0.99	0.0029	2.00	36.61
7	15.339	17.191	31.283	20.30	1.155	0.931	0.99	0.0033	2.00	34.56
8	17.044	18.236	31.683	20.26	1.202	0.912	0.99	0.0036	2.00	33.17
9	18.742	19.292	32.110	20.21	1.261	0.891	0.98	0.0039	2.00	31.83
10	20.429	20.367	32.875	20.09	1.347	0.862	0.98	0.0113	2.00	29.43
11	20.848	20.640	33.244	20.06	1.382	0.851	0.97	0.0129	0.50	28.94
12	21.266	20.915	33.653	20.04	1.425	0.838	0.97	0.0160	0.50	28.57
13	21.681	21.194	34.177	19.99	1.476	0.823	0.97	0.0211	0.50	27.87
14	22.092	21.477	34.871	19.90	1.539	0.806	0.97	0.0277	0.50	26.64
15	22.500	21.767	35.786	19.74	1.618	0.786	0.96	0.0368	0.50	24.98
16	22.903	22.063	37.001	19.55	1.716	0.763	0.95	0.0487	0.50	23.31
17	23.298	22.370	38.622	19.34	1.840	0.737	0.95	0.0657	0.50	21.79
18	23.683	22.689	40.761	19.09	2.002	0.707	0.94	0.0840	0.50	20.30
19	24.053	23.025	43.498	18.75	2.216	0.672	0.93	0.1087	0.50	18.57
20	24.405	23.380	47.137	18.25	2.504	0.632	0.93	0.1493	0.50	16.55
21	24.728	23.762	52.329	17.37	2.905	0.587	0.92	0.2210	0.50	13.78
22	25.004	24.179	60.757	15.41	3.489	0.535	0.93	0.3924	0.50	9.59
23	25.169	24.650	80.545	9.59	4.397	0.477	1.09	1.1307	0.50	3.10

RAY STOPPED
TEST CASE 32 PERIOD = 4.0SEC. ANGLE= 30.0DEG.X= 6.00Y= 9.30

N	X	Y	A	C	BETA	COEF RFTN	SHOAL COEF	CURV	S	DPTH
1	6.000	9.300	30.000	20.45	1.000	1.000	1.00	0.0004	4.00	44.06
2	7.732	10.301	30.051	20.45	1.000	1.000	1.00	0.0005	2.00	43.94
3	9.462	11.303	30.104	20.44	1.000	1.000	1.00	0.0002	2.00	42.83
4	11.192	12.307	30.123	20.44	1.000	1.000	1.00	0.0001	2.00	42.52
5	12.922	13.310	30.137	20.44	1.000	1.000	1.00	0.0000	2.00	42.64
6	14.652	14.315	30.140	20.44	1.002	0.999	1.00	0.0007	2.00	42.15
7	16.380	15.321	30.264	20.41	1.008	0.996	1.00	0.0014	2.00	40.02
8	18.106	16.332	30.471	20.37	1.018	0.991	0.99	0.0022	2.00	37.13
9	19.828	17.350	30.706	20.31	1.030	0.985	0.99	0.0015	2.00	34.91
10	21.546	18.373	30.867	20.28	1.040	0.981	0.98	0.0015	2.00	33.79
11	23.260	19.403	31.105	20.23	1.052	0.975	0.98	0.0029	2.00	32.31
12	24.970	20.441	31.432	20.18	1.069	0.967	0.98	0.0022	2.00	31.05
13	26.673	21.490	31.844	20.08	1.091	0.957	0.97	0.0061	2.00	29.28
14	27.520	22.021	32.245	20.04	1.111	0.949	0.97	0.0071	1.00	28.51

15	28.364	22.557	32.667	19.99	1.137	0.938	0.97	0.0074	1.00	27.91
16	29.203	23.101	33.201	19.91	1.176	0.922	0.96	0.0124	1.00	26.76
17	29.621	23.376	33.586	19.86	1.203	0.912	0.96	0.0135	0.50	26.25
18	30.036	23.654	33.990	19.84	1.236	0.899	0.96	0.0147	0.50	25.98
19	30.450	23.936	34.478	19.77	1.277	0.885	0.96	0.0204	0.50	25.29
20	30.860	24.221	35.200	19.63	1.327	0.868	0.95	0.0313	0.50	23.92
21	31.266	24.513	36.293	19.41	1.391	0.848	0.95	0.0458	0.50	22.23
22	31.665	24.815	37.846	19.08	1.474	0.824	0.94	0.0633	0.50	20.21
23	32.054	25.128	39.858	18.72	1.581	0.795	0.93	0.0758	0.50	18.42
24	32.431	25.457	42.276	18.36	1.718	0.763	0.93	0.0944	0.50	16.96
25	32.791	25.804	45.644	17.69	1.896	0.726	0.92	0.1501	0.50	14.70
26	33.122	26.179	51.534	16.45	2.149	0.682	0.92	0.2796	0.50	11.59
27	33.393	26.599	62.640	14.04	2.574	0.623	0.95	0.5250	0.50	7.49
28	33.514	27.084	89.359	6.89	3.342	0.547	1.25	1.5308	0.50	1.53

RAY STOPPED

TEST CASE 33 PERIOD = 4.0SEC. ANGLE= 30.0DEG.X= 7.00Y= 7.90

N	X	Y	A	C	BETA	COEF RFTN	SHOAL COEF	CURV	S	DPTH
1	7.000	7.900	30.000	20.47	1.000	1.000	1.00	0.0003	4.00	46.95
2	8.732	8.901	30.037	20.47	1.001	1.000	1.00	0.0003	2.00	46.19
3	10.463	9.902	30.066	20.46	1.002	0.999	1.00	0.0004	2.00	44.73
4	12.193	10.905	30.108	20.44	1.004	0.998	1.00	0.0003	2.00	42.94
5	13.923	11.908	30.137	20.44	1.005	0.997	1.00	-0.0000	2.00	42.04
6	15.653	12.913	30.136	20.43	1.006	0.997	1.00	-0.0002	2.00	41.96
7	17.383	13.916	30.115	20.44	1.006	0.997	1.00	0.0001	2.00	42.63
8	19.112	14.921	30.176	20.42	1.009	0.995	1.00	0.0011	2.00	40.99
9	20.840	15.929	30.359	20.38	1.016	0.992	0.99	0.0021	2.00	38.00
10	22.563	16.943	30.604	20.33	1.024	0.988	0.99	0.0019	2.00	35.33
11	24.283	17.964	30.790	20.30	1.029	0.986	0.98	0.0012	2.00	34.23
12	26.000	18.991	30.954	20.26	1.036	0.983	0.98	0.0020	2.00	33.01
13	27.712	20.023	31.214	20.21	1.046	0.978	0.98	0.0024	2.00	31.78
14	29.420	21.064	31.525	20.13	1.060	0.971	0.97	0.0031	2.00	30.21
15	31.121	22.116	31.912	20.06	1.077	0.963	0.97	0.0035	2.00	28.95
16	32.815	23.180	32.389	19.97	1.101	0.953	0.96	0.0051	2.00	27.60
17	33.657	23.718	32.740	19.92	1.118	0.946	0.96	0.0073	1.00	26.93
18	34.496	24.262	33.175	19.86	1.143	0.935	0.96	0.0073	1.00	26.16
19	35.331	24.814	33.726	19.78	1.178	0.922	0.96	0.0135	1.00	25.37
20	35.746	25.093	34.205	19.69	1.203	0.912	0.95	0.0200	0.50	24.47
21	36.157	25.376	34.868	19.58	1.236	0.899	0.95	0.0263	0.50	23.49
22	36.565	25.665	35.720	19.47	1.280	0.884	0.95	0.0334	0.50	22.72
23	36.969	25.961	36.844	19.34	1.339	0.864	0.94	0.0465	0.50	21.80
24	37.364	26.267	38.484	19.06	1.419	0.839	0.93	0.0706	0.50	20.09
25	37.749	26.586	41.052	18.50	1.532	0.808	0.93	0.1130	0.50	17.51
26	38.114	26.928	45.200	17.52	1.696	0.768	0.92	0.1835	0.50	14.20
27	38.444	27.304	52.173	15.81	1.944	0.717	0.92	0.3198	0.50	10.30
28	38.703	27.731	65.306	12.41	2.346	0.653	0.98	0.6438	0.50	5.55

RAY STOPPED

SHORELINE COORDINATES

1	1.000	16.048
2	2.000	16.500
3	3.000	17.000

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  4                    3.000  17.000
  5                    4.000  17.464
  6                    5.000  17.667
  7                    5.588  18.000
  8                    6.000  18.318
  9                    7.000  18.600
 10                    8.000  19.000
 11                    8.000  19.000
 12                    9.000  19.444
 13                   10.000  19.643
 14                   11.000  20.000
 15                   11.000  20.000
 16                   12.000  20.400
 17                   13.000  20.783
 18                   14.000  21.000
 19                   14.000  21.000
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      (INTERIM CARDS OMITTED)
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 52                   40.000  28.400
 53                   41.000  28.520
 54                   42.000  28.643
 55                   43.000  28.800
 56                   44.000  29.000
 57                   44.000  29.000
 58                   45.000  29.400
 59                   46.000  29.556
 60                   47.000  29.667
 61                   48.000  29.815
 62                   49.000  30.000
 63                   49.000  30.000
 64                   50.000  30.400
    END SHORELINE SEGMENT
ALL DONE.
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